Mean-Variance versus Naïve Diversification: The Role of Mispricing

Abstract

We compare the equal-weight naïve 1/N portfolio with mean-variance strategies from the perspective of mispricing (alpha) and provide three new findings. First, we analytically show that the 1/N rule approaches the *ex ante* mean-variance efficient portfolio in the absence of mispricing. With mispricings, mean-variance strategies can overcome the difficulty brought by the imprecise parameter estimates and outperform 1/N by exploiting the mispricing. Second, with mispricings the 1/N rule is unlikely to outperform mean-variance strategies even when N is large, since mean-variance strategies have more opportunities to exploit mispricings. Third, minimum-variance strategies do not exploit mispricings and underperform the 1/N rule.

Keywords: finance, portfolio choice, mean-variance, 1/N naïve diversification, mispricing

JEL Classification: C44, D81, G11, G12

1. Introduction

Mean-variance analysis is the cornerstone of modern finance. Markowitz (1952) provides a rigorous framework to consider the risk-return tradeoff, and a methodology to construct optimal portfolios. Although the mean-variance analysis is used pervasively in the academia, the main difficulty in its practical implementations stems from the estimation error or parameter uncertainty problem (Brandt, 2009). Good estimates of the first and second moments are necessary for mean-variance optimization to provide reasonable portfolio weights. An alternative to the mean-variance framework is the naïve equal-weight portfolio investing 1/N of total wealth in each of the N assets, which can be found in the ancient Babylonian Talmud 1500 years ago and has been observed for individual investors in modern times (Benartzi and Thaler, 2001; Huberman and Jiang, 2006; Brown et al., 2007). The 1/N rule does not require parameter estimation and it has been shown that the mean-variance strategies cannot beat the 1/N rule in a strand of literature including, among others, DeMiguel et al. (2009), casting doubts upon the practical usefulness of the Markowitz framework.

We evaluate the performance of the 1/N rule relative to a broad set of mean-variance strategies and provide three new findings. We present an analytical expression to understand the performance of the 1/N rule. If the Capital Asset Pricing Model (CAPM) holds, for instance, the market portfolio coincides with the *ex ante* tangency portfolio, which has the highest possible Sharpe ratio. With low idiosyncratic volatility relative to market volatility and a large number of assets, the Sharpe ratio of the 1/N rule approaches that of the market portfolio. In this case, the 1/N rule is likely to outperform sample-based mean-variance strategies, which are plagued by estimation errors. Our analytical expression provides an explanation for the excellent performance of the 1/N rule in DeMiguel et al. (2009) without resorting to simulations.

We show that the mean-variance strategies can beat the simple 1/N rule when the CAPM does not hold, even with a large N. Deviations relative to the CAPM (mispricings or alphas) imply the market portfolio is no longer mean-variance optimal. Whereas the Sharpe ratio of the 1/N rule still approaches that of the market portfolio, the mean-variance strategies can exploit the mispricing to form portfolios with higher Sharpe ratios. Holding N constant, for sufficiently large mispricings, mean-variance strategies will outperform the 1/N rule. As the number of assets N increases, there is a tradeoff between precisely estimating the covariance matrix and exploiting mispriced assets. Our simulations show that, given sufficiently large deviations from the cross-sectional asset-pricing model, an increase in the number of securities will cause mean-variance strategies to outperform the 1/N rule. This result overturns the findings in DeMiguel et al. (2009)[1] but is consistent with Huberman and Jiang (2006)[2]. Although we use the CAPM as a benchmark model in our analysis, our results hold under more general models including the Fama and French (1992, 1993) and Carhart (1997) models.

Not all mean-variance strategies are able to beat the 1/N rule. Estimation errors in the sample means have a greater influence on the performance of mean-variance strategies than the ones in the sample covariance matrix. As a result, the literature has shifted attention from mean-variance strategies to minimum-variance strategies (Green and Hollifield, 1992; Jagannathan and Ma, 2003; Ledoit and Wolf, 2003; DeMiguel et al., 2009)[3]. However, Wang et al.

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[1]DeMiguel et al. (2009) note "*What is N? That is, for what number and kind of assets does the 1/N strategy outperform other optimizing portfolio models? The results show that the naive 1/N strategy is more likely to outperform the strategies from the optimizing models when: (i) N is large, because this improves the potential for diversification, even if it is naive, while at the same time increasing the number of parameters to be estimated by an optimizing model; (ii) the assets do not have a sufficiently long data history to allow for a precise estimation of the moments.*"

[2]Huberman and Jiang (2006) note in their abstract that "*Records of over half a million participants in more than 600 401(k) plans indicate that participants tend to allocate their contributions evenly across the funds they use, with the tendency weakening with the number of funds used*".

[3]Minimum-variance strategies can be seen as a special case of the mean-variance strategies. For instance,

(2015) suggest that it is difficult to find a strategy under the minimum-variance framework that reliably outperforms the naïve 1/N strategy. In our simulations, the 1/N rule consistently outperforms several variations of the minimum-variance portfolio, including the true minimum-variance portfolio based on population moments. This is not surprising, as the minimum-variance portfolios are designed to have the lowest feasible variance, but not necessarily the highest Sharpe ratio.

We confirm our simulation results through an empirical investigation using the size and book-to-market portfolios, the Fama-French factors, and the industry portfolios. Although DeMiguel et al. (2009) find that the mean-variance strategies can hardly beat the 1/N rule, their data is from July 1963 to November 2004, and excludes the Global Financial Crisis (GFC) in the late 2000s during which mispricings may have been the largest. Using an extended sample from July 1963 through December 2015, we find that a number of mean-variance strategies are able to outperform the 1/N rule.

The central intuition for our findings is based on the tradeoff between the exploitation of mispricing and sampling variation in estimated parameters when comparing mean-variance strategies against the 1/N rule. In the absence of mispricing, estimation errors cause the mean-variance strategies to under-perform the 1/N rule. Mispricings provide mean-variance strategies an advantage over the 1/N rule in that mean-variance strategies can benefit from mispricing through intelligently changing the portfolio weights to increase expected returns. This advantage and the disadvantage from estimation errors both increase with the number of investable assets, and the former dominates given sufficiently large mispricings. Such a tradeoff does not apply to minimum-variance strategies, which do not exploit mispricing to increase

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DeMiguel et al. (2009) note "*Also, although this strategy does not fall into the general structure of mean-variance expected utility, its weights can be thought of as a limiting case of Equation (3), if a mean-variance investor either ignores expected returns or, equivalently, restricts expected returns so that they are identical across all assets*".

expected returns. By construction, the minimum-variance portfolios are only concerned about risk and ignore the information from the expected returns.

Our paper most closely relates to DeMiguel et al. (2009), Tu and Zhou (2011) and Wang et al. (2015). DeMiguel et al. (2009) compare the 1/N rule against mean-variance strategies and find that the mean-variance strategies can hardly beat the 1/N rule. We uncover the important role of the zero mispricing in their study with a closed-form expression, and overturn their result that the mean-variance strategies cannot beat the 1/N rule when N is large by introducing deviations from the cross-sectional model. Whereas Tu and Zhou (2011) advocate the better performance of their newly proposed combination rules under non-zero mispricing, we ask if other mean-variance strategies also outperform and investigate the size of mispricing required for outperformance relative to the 1/N rule. Wang et al. (2015) suggest that the minimum-variance strategy cannot outperform the naïve 1/N strategy in a two-asset case when hedging the underlying returns with futures. We extend their asset allocation exercise to more assets and confirm their findings in a more general case.

Our first result from simulationsthat, mean-variance strategies can beat the simple 1/N rule in the presence of mispricing was originally suggested by Tu and Zhou (2011). We include this result here for two reasons. First, it provides a very useful springboard for our two other contributions, namely the analysis of the impact of the number of investable assets (N) and the performance of the minimum-variance strategies. Second, we are able to offer a theoretical reasoning framework with some closed-form results which document all the factors attributing to the excellent performance of the 1/N rule in DeMiguel et al. (2009). As part of our theoretical reasoning framework, we show that, there is a tradeoff between accurately estimating the covariance matrix and exploiting mispricing when comparing the mean-variance strategies against the 1/N rule a result that is interesting in its own right and that is new to the literature.

Our paper also relates to the broader literature comparing mean-variance strategies against the 1/N rule. A strand of literature examines this question by taking the perspective of the mean-variance strategies, using the 1/N rule as the benchmark. This strand of literature commonly attributes the under-performance of the mean-variance strategies to estimation errors: Brown (1976, 1979), Jobson and Korkie (1980), Michaud (1989), Jorion (1992), Duchin and Levy (2009), and Moorman (2014) are some examples that do so. In contrast, a small but growing literature, to which our paper belongs, investigates this question from the perspective of the 1/N rule (Pflug et al., 2012). While there is a large literature aiming to beat the 1/N rule through developing more advanced strategies (Tu and Zhou, 2011; Kirby and Ostdiek, 2012), we seek to understand the different environments in which mean-variance strategies would be useful.

Finally, our paper also connects to the literature about active management and passive management in the markets with the various extents of efficiency. Based on our results, we conjecture that mean-variance strategies should be more successful in emerging markets (EMs) than in developed markets (DMs) as it is more likely to have larger mispricing in EMs than in DMs. We do not empirically test this conjecture in this paper, as there are ample evidence presented in a long line of literature such as Harvey (1995), Morck et al. (2000), Van der Hart et al. (2003), and Griffin et al. (2010). A recent example is Dyck et al. (2013), who use a proprietary data set from 1993 to 2008 and find that "*For sophisticated institutional investors, active management outperforms passive management by more than 180 bps per year in emerging markets and by about 50 bps in EAFE markets over the 1993 to 2008 period. In U.S. markets, active management underperforms. Consistent with these patterns in returns, institutions use active management more frequently in non-U.S. markets, particularly emerging marketsOverall, our results suggest that the value of active management depends on the efficiency of the underlying market and the sophistication of the investor.*"

The paper proceeds as follows. In section 2, we put forward an analytic expression to understand the relative performance of the 1/N rule versus the mean-variance strategies. Section 3 describes the portfolio rules we consider. Section 4 presents the results of our portfolio rules from simulations. Section 5 affirms our simulation results using real data. Section 6 discusses the robustness of our simulation setting. Section 6 concludes.

2. The Role of Mispricing

In this section, we derive analytical expressions to understand the relative performance of the 1/N rule versus the mean-variance portfolios. Suppose there is a risk-free asset, and N risky assets. Let R_{mt} denote the excess return on the market portfolio over the risk-free rate, and $\mathbf{R}_t$ be the $N \times 1$ vector of excess returns of the risky assets. Suppose the Capital Asset Pricing Model (CAPM) describes the cross-section of average returns. $\mathbf{R}_t$ can be written in the market model of Sharpe (1963, 1964):

$$\mathbf{R}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} R_{mt} + \boldsymbol{\epsilon}_t \tag{1}$$

where $\boldsymbol{\alpha}$ is the $N \times 1$ vector of intercepts (mispricings), $\boldsymbol{\beta}$ is the $N \times 1$ vector of betas, and $\boldsymbol{\epsilon}_t$ is the $N \times 1$ vector of errors. Suppose the market model is the true factor model, and $\boldsymbol{\epsilon}_t$ has a diagonal covariance matrix Σ_ϵ whose diagonal elements are $[\Sigma_\epsilon]_{ii} = \sigma_i$. Let $\mathbb{E}[R_{mt}] = \mu_m$ and $\text{Var}(R_{mt}) = \sigma_m^2$. The mean and covariance matrix of the risky assets are as follows:

$$\boldsymbol{\mu} = \mathbb{E}[\mathbf{R}_t] = \boldsymbol{\alpha} + \boldsymbol{\beta}\mu_m \tag{2}$$

$$\Sigma = \mathbb{E}[(\mathbf{R}_t - \boldsymbol{\mu})(\mathbf{R}_t - \boldsymbol{\mu})'] = \boldsymbol{\beta}\boldsymbol{\beta}'\sigma_m^2 + \Sigma_\epsilon \tag{3}$$

Treynor and Black (1973) show that Sharpe ratio of the mean-variance optimal portfolio p, sr_p, has the following relationship with the Sharpe ratio of the market portfolio sr_m and appraisal ratios sr_i of individual assets:

$$sr_p^2 = sr_m^2 + \sum_{i=1}^{N} sr_i^2 \tag{4}$$

where $sr_i = \frac{\alpha_i}{\sigma_i}$ is the mispricing relative to the CAPM over the standard deviation of the mispricing.

2.1. When the CAPM Holds

Suppose the CAPM holds, the mispricing $\boldsymbol{\alpha} = \mathbf{0}$. It follows from Equation (4) that the second term on the right-hand side is zero and the mean-variance optimal portfolio is the market portfolio. We can express the Sharpe ratio of the 1/N rule as follows:

$$sr(1/N) = \frac{\mathbf{1}_N'\boldsymbol{\mu}}{\sqrt{\mathbf{1}_N'\Sigma\mathbf{1}_N}} = \frac{\mathbf{1}_N'(\boldsymbol{\alpha} + \boldsymbol{\beta}\mu_m)}{\sqrt{\mathbf{1}_N'(\boldsymbol{\beta}\boldsymbol{\beta}'\sigma_m^2 + \Sigma_\epsilon)\mathbf{1}_N}} \tag{5}$$

For $\boldsymbol{\alpha} = \mathbf{0}$, Equation (5) can be written as the following:

$$sr(1/N) = \frac{\mu_m}{\sigma_m} \frac{1}{\sqrt{1 + \frac{\bar{\sigma_\epsilon}^2}{N\sigma_m^2\bar{\beta}}}} \tag{6}$$

where $\bar{\sigma_\epsilon} = \sum_{i=1}^{N} \sigma_i/N$ is the average of the idiosyncratic volatilities and $\bar{\beta} = \sum_{i=1}^{N} \beta_i/N$ is the average of the betas in the market. In a well-diversified portfolio, $\bar{\beta}$ approximately approaches 1. The Sharpe ratio of the 1/N portfolio and the market portfolio differs by a scale factor $\frac{1}{\sqrt{1+\frac{\bar{\sigma_\epsilon}^2}{N\sigma_m^2}}}$.

If the CAPM holds, the market is the mean-variance efficient portfolio with the highest Sharpe ratio. It follows from Equation (6) that the Sharpe ratio of the 1/N portfolio approaches that of the market portfolio under two conditions: (i) The average idiosyncratic volatility is low relative to the market volatility, and (ii) The number of assets, N, is large. The first condition means that, if portfolios are already well-diversified, the advantage of the mean-variance portfolios relative to the 1/N rule is small. The second condition states that, if the number of assets is large, mean-variance analysis needs to estimate many parameters, and the potential estimation errors make the mean-variance portfolios unattractive relative to the 1/N rule.

These conditions help understand the findings in DeMiguel et al. (2009), who claim that the 1/N rule performs better selecting portfolios rather than individual assets. Portfolios tend to have lower idiosyncratic risk compared to individual securities, which according to our first condition, makes the 1/N rule more attractive relative to the mean-variance analysis. DeMiguel et al. (2009) also find that the 1/N rule works better when N is large, which corresponds to our second condition. Our analytical expression offers a simple way to understand these results without simulations.

2.2. When the CAPM does not Hold

If the mispricing $\boldsymbol{\alpha} \neq \mathbf{0}$, the CAPM does not hold, and the market portfolio is not necessarily mean-variance efficient. Under the two conditions from the previous section, the Sharpe ratio of the 1/N portfolio still approaches that of the market portfolio. Since the market portfolio no longer has the highest Sharpe ratio, it is possible for mean-variance strategies to outperform 1/N. The question becomes a quantitative one: How large do the CAPM deviations, the mispricing $\boldsymbol{\alpha}$, have to be for the mean-variance portfolios to outperform the 1/N rule? We offer some intuitions here for the relative performance of the 1/N rule versus the mean-variance strategies, and make the claims more concrete in the next section.

Condition (i) does not offer the 1/N rule such a significant advantage when the CAPM fails to hold. Although the 1/N portfolio still approaches the market portfolio under Condition (i) when the CAPM does not hold, the market portfolio is no longer optimal as it does not make use of the non-zero mispricings at all. In contrast, the mean-variance portfolios are able to exploit the non-zero mispricings by adjusting the portfolio weights according to the sign and magnitude of individual mispricing α_i , which renders the mean-variance portfolios an advantage over the 1/N rule.

Condition (ii) also fails to help the 1/N rule in the presence of a non-zero mispricing $\boldsymbol{\alpha}$. When the number of assets N is large, the performance of the mean-variance portfolios relative to the 1/N rule hinges on a tradeoff. On the one hand, the mean-variance analysis must estimate many parameters in the covariance matrix. On the other hand, conditional on a good covariance matrix estimate, mean-variance strategies can optimally choose weights in accordance with the mispricing $\boldsymbol{\alpha}$ to improve the expected return of the portfolio. The former makes it more difficult for the mean-variance portfolios to outperform the 1/N rule, whereas the latter offers the mean-variance portfolios an advantage, and it becomes an empirical question which effect is stronger.

3. Portfolio Rules

We examine a variety of portfolios construction rules used in the literature to compare the performance of the mean-variance strategies with the one of the 1/N rule. We do not consider the shortsale-constrained portfolios in DeMiguel et al. (2009), as Jagannathan and Ma (2003) have shown that imposing such a constraint is equivalent to shrinking the covariance matrix which improves the results for the mean-variance strategies. We want to focus on the role of mispricing, and understand how the mean-variance strategies could outperform the 1/N rule without relying on superior covariance matrix estimates. Including shortsale constraints in our studies is likely to improve the performance of the mean-variance strategies relative to the 1/N rule, which will confound our understanding of the role of mispricing. We also do not evaluate the models in Best and Grauer (1992), Chan et al. (1999), Ledoit and Wolf (2004a,b), and Jagannathan and Ma (2003) for the same reason.

Following DeMiguel et al. (2009), we report results neither for the purely statistical approach relying on Bayesian diffuse-priors (Barry, 1974; Bawa et al., 1979), nor for the multi-prior robust portfolio rules such as the one in Garlappi et al. (2007). Regarding Tu and Zhou (2011), we consider the optimal combination of the 1/N rule and the sample tangency portfolio, and the optimal combination of the 1/N rule and the portfolio in Kan and Zhou (2007), because they are the only ones analytically tractable and considered in more recent literature such as Moorman (2014). We also exclude portfolios rules that do not require optimization and covariance matrix inversion, the two most notable characteristics of the mean-variance portfolios, such as the ones in Kirby and Ostdiek (2012). As a result, we focus on 12 portfolios rules in total as described below.

3.1. Naïve Diversification ("naïve")

Naïve diversification calls for equal dollar amounts allocated among the N risky assets:

$$\mathbf{x}_e = \mathbf{1}_N / N \tag{7}$$

This is an equal-weight portfolio rebalanced constantly. No parameter estimation or portfolio optimization is necessary. In theory, naïve diversification deviates from the mean-variance optimal portfolio and thus has suboptimal performances. In practice, its performance depends on the tradeoff between its deviation from the *ex ante* tangency portfolio and its immunity to estimation risk. We call this strategy the "naïve" portfolio.

3.2. The Tangency Portfolio ("True" and "Sample")

From mean-variance theory, the tangency portfolio provides the highest Sharpe ratio of any feasible investment combination. If an investor invests $\mathbf{x}$ in N risky assets, and $(1 - \mathbf{1}_N' \mathbf{x})$

in the risk-free asset, the relative weights in the investor's portfolio with risky assets is

$$\mathbf{w} = \frac{\mathbf{x}}{|\mathbf{1}_N'\mathbf{x}|} \tag{8}$$

where the absolute value guarantees the relative weights $\mathbf{w}$ have the same sign as $\mathbf{x}$.

Mean-variance optimization can be motivated from a quadratic utility function:

$$U(\mathbf{x}) = \mathbf{x}'\boldsymbol{\mu} - \frac{\gamma}{2}\mathbf{x}'\Sigma\mathbf{x} \tag{9}$$

where γ is the risk-aversion coefficient. The mean-variance optimal portfolio is the tangency portfolio, with weights:

$$\mathbf{x}^* = \frac{\Sigma^{-1}\boldsymbol{\mu}}{\gamma} \tag{10}$$

To operationalize the tangency portfolio, we need estimates for $\boldsymbol{\mu}$ and Σ. The following sample analogues are often used:

$$\hat{\boldsymbol{\mu}} = \frac{1}{T}\sum_{1}^{T}\mathbf{R}_t, \quad \hat{\Sigma} = \frac{1}{T-N-2}\sum_{1}^{T}(\mathbf{R}_t - \hat{\boldsymbol{\mu}})(\mathbf{R}_t - \hat{\boldsymbol{\mu}})' \tag{11}$$

Sometimes T or $T-1$ will be used in $T-N-2$ for $\hat{\Sigma}$. All three are unbiased asymptotically and do not differ much for a large T.

We examine two constructions of the tangency portfolio. The *ex ante* (true) tangency portfolio is obtained by substituting in the true population mean returns and covariance matrix into Equation (10). The sample tangency portfolio uses estimates from Equation (11) in place of population moments. We call these the "True" and "Sample" portfolios.

3.3. The Global Minimum-Variance Portfolio ("min_True" and "min_Sample")

The global minimum-variance portfolio is the portfolio of risky assets with the lowest possible variance. The set of portfolio weights is the solution to the following problem:

$$\underset{\mathbf{x}}{\text{minimize}} \quad \mathbf{x}'\Sigma\mathbf{x} \qquad \text{subject to} \quad \mathbf{1}_N'\mathbf{x} = 1 \tag{12}$$

where Σ is the covariance matrix of risky assets. The solution is

$$\mathbf{x}^{min} = \frac{\Sigma^{-1}\mathbf{1}_N}{\mathbf{1}_N'\Sigma^{-1}\mathbf{1}_N} \tag{13}$$

If we had the population covariance matrix, Equation (13) calculates the *ex ante* global minimum-variance portfolio, "min_True". Using the sample covariance matrix yields the *ex post* global minimum-variance portfolio, "min_Sample".

The sample global minimum-variance portfolio differs from the sample mean-variance portfolio in that it only requires estimating the covariance matrix. Therefore, it is less prone to estimation errors. Merton (1980) argues that the covariance matrix can be more precisely estimated compared to sample means. However, the global minimum-variance portfolio does not possess the desired property of the *ex ante* highest Sharpe ratio like the tangency portfolio.

3.4. Jorion's (1986) Bayes-Stein Shrinkage Estimators ("Jorion")

Jorion (1986) reports that the tangency portfolio based on the Bayes-Stein estimators outperforms the global minimum-variance portfolio in terms of expected utility loss, using the following estimators for the mean vector and the covariance matrix:

$$\hat{\boldsymbol{\mu}}^{bs} = (1-\hat{\delta})\hat{\boldsymbol{\mu}} + \hat{\delta}\hat{\boldsymbol{\mu}}^{min}\mathbf{1}_N \tag{14}$$

$$\hat{\Sigma}^{bs} = \left(1 + \frac{1}{T+\hat{\tau}}\right)\hat{\Sigma} + \frac{\hat{\tau}}{T(T+1+\hat{\tau})}\frac{\mathbf{1}_N\mathbf{1}_N'}{\mathbf{1}_N'\hat{\Sigma}^{-1}\mathbf{1}_N} \tag{15}$$

where

$$\hat{\tau} = \frac{T\hat{\delta}}{1-\hat{\delta}} \tag{16}$$

$$\hat{\delta} = \frac{N+2}{N+2+T(\hat{\boldsymbol{\mu}}-\hat{\boldsymbol{\mu}}^{min})'\hat{\Sigma}^{-1}(\hat{\boldsymbol{\mu}}-\hat{\boldsymbol{\mu}}^{min})} \tag{17}$$

The Bayes-Stein mean estimate $\hat{\boldsymbol{\mu}}^{bs}$ shrinks the sample average $\hat{\boldsymbol{\mu}}$ towards $\hat{\boldsymbol{\mu}}^{min}$, the return on the sample global minimum-variance portfolio. We use the estimates from Equations (14) and (15) as inputs for the tangency portfolio in Equation (10). We call this portfolio "Jorion".

3.5. Bayesian Data-and-Model ("dm")

Pástor (2000) and Pástor and Stambaugh (2000) propose a Bayesian data-and-model portfolio construction which allows the investor to combine his confidence in an asset pricing model with the information from sample data. Suppose asset returns are generated by the market model as in Equation (1). Let $\left(\tilde{\boldsymbol{\beta}}, \tilde{\Sigma}_\epsilon\right)$ and $\left(\hat{\boldsymbol{\beta}}, \hat{\Sigma}_\epsilon\right)$ denote the maximum likelihood estimates for $(\boldsymbol{\beta}, \Sigma_\epsilon)$ from Equation (1) with and without imposing the restriction $\boldsymbol{\alpha} = \mathbf{0}$. The investor would use these estimators if he had no prior belief about the asset pricing model. A Bayesian investor who has a certain degree of confidence in the model would specify a prior distribution on the mispricings $\boldsymbol{\alpha}$.

Assuming a normal prior on $\boldsymbol{\alpha}$:

$$\mathrm{p}(\boldsymbol{\alpha}|\Sigma_\epsilon) \sim \mathrm{N}(0, \tau\Sigma_\epsilon) \tag{18}$$

where τ is the precision of the prior belief in the CAPM. The priors on $\boldsymbol{\beta}$, Σ_ϵ, μ_m, and σ_m^2 are assumed to be independent and non-informative.

Wang (2005) shows that the Bayesian estimators of expected returns and covariance matrix can be written as follows:

$$\hat{\boldsymbol{\mu}}^{dm} = \hat{\delta}\tilde{\boldsymbol{\beta}}\hat{\boldsymbol{\mu}}_m + \left(1-\hat{\delta}\right)\hat{\boldsymbol{\mu}} \tag{19}$$

$$\Sigma^{dm} = \kappa\bar{\boldsymbol{\beta}}\bar{\boldsymbol{\beta}}'\hat{\sigma}_m^2 + h\left(\hat{\delta}\tilde{\phi} + \left(1-\hat{\delta}\right)\hat{\phi}\right)\left(\hat{\delta}\tilde{\Sigma}_\epsilon + \left(1-\hat{\delta}\right)\hat{\Sigma}_\epsilon\right) \tag{20}$$

where

$$\hat{\delta} = \frac{1}{1+T\tau/\left(1+\hat{S}^2\right)}$$

$$\hat{S}^2 = \hat{\mu}_m^2/\hat{\sigma}_m^2, \quad \bar{\boldsymbol{\beta}} = \hat{\delta}\tilde{\boldsymbol{\beta}} + \left(1-\hat{\delta}\right)\hat{\boldsymbol{\beta}}$$

$$\tilde{\phi} = \frac{T(T-2)+1}{T(T-3)} - \frac{4\hat{S}^2}{T(T-3)(1+\hat{S}^2)}$$

$$\hat{\phi} = \frac{(T-2)(T+1)}{T(T-3)}, \quad \kappa = \frac{T+1}{T-3}, \quad h = \frac{T}{T-N-1}$$

The mean estimate $\hat{\boldsymbol{\mu}}^{dm}$ shrinks the sample average $\hat{\boldsymbol{\mu}}$ towards $\tilde{\boldsymbol{\beta}}\hat{\boldsymbol{\mu}}_m$, the maximum likelihood estimator obtained under the CAPM restriction. The shrinkage parameter $\hat{\delta}$ is a decreasing function of τ. As τ approaches zero, the investor increasingly believes the CAPM holds exactly and estimates $\boldsymbol{\mu}$ with $\tilde{\boldsymbol{\beta}}\hat{\boldsymbol{\mu}}_m$. As τ grows large, the investor has little confidence in the CAPM and estimates $\boldsymbol{\mu}$ with the sample average $\hat{\boldsymbol{\mu}}$. The estimator for the covariance matrix works similarly. We show the results from a weak belief in the CAPM with a large τ that we call the "dm" portfolio but our results also hold when we consider a strong belief.

3.6. MacKinlay and Por (2000) ("MacKinlay-Por")

MacKinlay and Pástor (2000) assume returns have an exact factor structure but one of the factors is unobserved. This implies a restriction that links the mispricing and the residual covariance matrix which they exploit to improve portfolio selection. Suppose returns $\mathbf{R}_t$ follow

Equation (1). If the market factor is unobservable, the model becomes the following:

$$\mathbf{R}_t = \boldsymbol{\alpha} + \boldsymbol{\nu}_t \tag{21}$$

$$\Sigma = \boldsymbol{\alpha}\boldsymbol{\alpha}' \frac{1}{sr_m^2} + \Phi \tag{22}$$

Assuming $\Phi = \sigma^2 I$, where I is a conforming identity matrix, MacKinlay and Pástor (2000) derive the tangency portfolio:

$$\mathbf{x}^{MP} = \frac{\boldsymbol{\mu}}{\mathbf{1}_N' \boldsymbol{\mu}} \tag{23}$$

We use the sample mean $\hat{\boldsymbol{\mu}}$ to form the "MacKinlay-Por" portfolio.

The assumption of an exact factor model may appear strong, but MacKinlay and Pástor (2000) show that the benefits of relaxing this assumption are small. Model misspecification is a big issue here. For our application, if the market model describes returns well, the MacKinlay and Pástor (2000) portfolio is likely to outperform. However, if the exact one-factor structure does not hold, this portfolio will not converge to the optimal one even as the sample size increases. This setup is similar to the Bayesian data-and-model portfolio with a strong belief in the CAPM.

3.7. Mixture of Minimum-Variance and the 1/N Portfolio ("ew-min")

Motivated by the difficulty in estimating expected returns, DeMiguel et al. (2009) consider a strategy that combines the minimum-variance portfolio with the 1/N rule. Covariances are relatively easier to estimate compared to expected returns, so one may want to make use of the covariance estimates but not necessarily the expected return estimates. The "ew-min" portfolio is given by the following

$$\mathbf{x}^{ew-min} = c\frac{1}{N}\mathbf{1}_N + d\hat{\Sigma}^{-1}\mathbf{1}_N \qquad \text{subject to} \quad \mathbf{1}_N'\mathbf{x}^{ew-min} = 1 \tag{24}$$

3.8. Kan and Zhou's (2007) Three-Fund Rule ("Kan-Zhou")

If the estimation errors of two risky portfolios are not perfectly correlated, one can reduce the estimation errors by combining them. Kan and Zhou (2007) propose using the sample global minimum-variance portfolio to reduce estimation risk for the sample tangency portfolio. Their portfolio can be expressed as the following:

$$\mathbf{x}^{KZ} = \frac{(T-N-1)(T-N-4)}{\gamma T(T-2)}\left[\eta\hat{\Sigma}^{-1}\hat{\boldsymbol{\mu}} + (1-\eta)\hat{\boldsymbol{\mu}}^{min}\hat{\Sigma}^{-1}\mathbf{1}_N\right] \tag{25}$$

$$\eta = \frac{\psi^2}{\psi^2 + N/T}$$

$$\psi^2 = \left(\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}^{min}\right)'\Sigma^{-1}\left(\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}^{min}\right)$$

Where $\hat{\boldsymbol{\mu}}^{min}$ is the excess return on the global minimum-variance portfolio.

Kan and Zhou (2007) call their portfolio the three-fund rule since the investor should allocation his wealth into three funds: The sample global minimum-variance portfolio, the sample tangency portfolio, and the risk-free asset. As the number of assets relative to time (N/T) grows, a larger fraction of wealth is allocated in the global minimum-variance portfolio as the tangency portfolio parameters become increasingly difficult to estimate.

We form the "Kan-Zhou" portfolio with the following estimates:

$$
\begin{aligned}
\hat{\eta} &= \frac{\hat{\psi}^2}{\hat{\psi}^2 + N/T} \\
\hat{\psi}^2 &= \frac{(T-N-1)\bar{\psi}^2 - (N-1)}{T} + \frac{2(\bar{\psi}^2)^{\frac{N-1}{2}}(1+\bar{\psi}^2)^{-\frac{T-2}{2}}}{TB_{\bar{\psi}^2/(1+\bar{\psi}^2)}((N-1)/2,(T-N+1)/2)} \\
\bar{\psi}^2 &= \left(\hat{\boldsymbol{\mu}} - \hat{\boldsymbol{\mu}}^{min}\right)' \hat{\Sigma}^{-1} \left(\hat{\boldsymbol{\mu}} - \hat{\boldsymbol{\mu}}^{min}\right)
\end{aligned}
\tag{26}
$$

where $\mathrm{B}_x(a,b) = \int_0^x y^{a-1}(1-y)^{b-1}dy$ is the incomplete beta function.

3.9. Combination of 1/N with the Sample Tangency Portfolio ("CML")

Tu and Zhou (2011) propose combining the 1/N rule with the sample tangency portfolio to reduce estimation risk. The resulting portfolio is as follows:

$$
\hat{\mathbf{x}}^{CML} = \frac{\hat{\pi}_2}{\hat{\pi}_1 + \hat{\pi}_2}\mathbf{x}_e + \frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_2}\hat{\mathbf{x}}^* \tag{27}
$$

where

$$
\begin{aligned}
\hat{\pi}_1 &= \mathbf{x}_e'\hat{\Sigma}\mathbf{x}_e - \frac{2}{\gamma}\mathbf{x}_e'\hat{\boldsymbol{\mu}} + \frac{1}{\gamma^2}\hat{\theta}^2 \\
\hat{\pi}_2 &= \frac{1}{\gamma^2}(c_1 - 1)\hat{\theta}^2 + \frac{c_1}{\gamma^2}\frac{N}{T} \\
c_1 &= \frac{(T-2)(T-N-2)}{(T-N-1)(T-N-4)} \\
\hat{\theta}^2 &= \frac{(T-N-2)\bar{\theta}^2 - N}{T} + \frac{2(\bar{\theta}^2)^{N/2}(1+\bar{\theta}^2)^{-\frac{T-2}{2}}}{N\mathrm{B}_{\frac{\bar{\theta}^2}{1+\bar{\theta}^2}}(N/2,(T-N)/2)} \\
\bar{\theta}^2 &= \hat{\boldsymbol{\mu}}'\hat{\Sigma}^{-1}\hat{\boldsymbol{\mu}}
\end{aligned}
\tag{28}
$$

where $\mathrm{B}_x(a,b)$ is the incomplete beta function as defined earlier. $\mathbf{x}_e = \mathbf{1}_N/N$ for the 1/N portfolio and $\hat{\mathbf{x}}^*$ is the sample tangency portfolio. We call this portfolio "CML".

3.10. Combination of 1/N with the Three-Fund Portfolio ("CKZ")

Tu and Zhou (2011) also combine the 1/N rule with the three-fund rule proposed by Kan and Zhou (2007). This portfolio has weights:

$$\hat{\mathbf{x}}^{CKZ} = \left(1 - \frac{\hat{\pi}_1 - \hat{\pi}_{13}}{\hat{\pi}_1 - 2\hat{\pi}_{13} + \hat{\pi}_3}\right)\mathbf{x}_e + \frac{\hat{\pi}_1 - \hat{\pi}_{13}}{\hat{\pi}_1 - 2\hat{\pi}_{13} + \hat{\pi}_3}\hat{\mathbf{x}}^{KZ} \tag{29}$$

where

$$\hat{\pi}_3 = \frac{\hat{\theta}^2}{\gamma^2} - \frac{1}{\gamma^2 c_1}\left(\hat{\theta}^2 - \frac{N\hat{\eta}}{T}\right)$$
$$\hat{\pi}_{13} = \frac{\hat{\theta}^2}{\gamma^2} - \frac{1}{\gamma}\mathrm{w}_e\hat{\boldsymbol{\mu}} + \frac{1}{\gamma c_1}\left(\hat{\eta}\mathrm{w}_e'\hat{\boldsymbol{\mu}} + (1-\hat{\eta})\hat{\boldsymbol{\mu}}^{min}\mathrm{w}_e'\mathbf{1}_N\right) - \frac{1}{\gamma}\left[\hat{\eta}\hat{\boldsymbol{\mu}}'\hat{\Sigma}^{-1}\hat{\boldsymbol{\mu}} + (1-\hat{\eta})\hat{\boldsymbol{\mu}}^{min}\hat{\boldsymbol{\mu}}'\hat{\Sigma}^{-1}\mathbf{1}_N\right]$$

where $\hat{\mathbf{x}}^{KZ}$ is given in Equation (25) and $\hat{\eta}$, $\hat{\pi}_2$, c_1, and $\hat{\theta}^2$ are given in (28). We call this portfolio "CKZ".

Table 1 provides a summary of all of the portfolios rules we consider. The 1/N rule is the only one that does not require parameter estimation.

4. Simulation

We simulate a panel of asset returns and form portfolios based on the rules described in the previous section. We evaluate their performance to determine the best portfolio rule.

4.1. Setup

We simulate monthly market returns R_{mt} by drawing from a normal distribution with an annualized mean of 8% and standard deviation of 16%. Individual securities are generated from a market model. $\boldsymbol{\beta}$ is drawn from a uniform distribution from 0.5 to 1.5, Unif$[0.5, 1.5]$. Idiosyncratic errors $\boldsymbol{\epsilon}$ are simulated from a multivariate normal distribution with zero mean and a diagonal covariance matrix. The diagonal elements are drawn from Unif$[0.1, 0.3]$ such that the cross-sectional average of annual volatility is 20%. Stemming from MacKinlay and Pástor (2000), all of these parameter values have also been used in DeMiguel et al. (2009) and Tu and Zhou (2011). For $\boldsymbol{\alpha}$, we either set it to zero (as in DeMiguel et al., 2009) or sample from Unif$[-i\%, i\%]$, where $i = 1, 2, 3, 4, 5$. We examine the robustness of our parameter choices in a later section.

We simulate $T = 1200$ months with 10, 25, or 50 securities. Let the estimation window be $L = \{120, 240, ..., 1200\}$ months used to estimate the portfolio weights in each portfolio rule. Let $s = \{1, 2, ..., 12\}$ denote the portfolio rules. We repeat our simulations 10000 times.

Let $\tilde{\mathbf{w}}^i_{s,L}$ be the weights for the portfolio rule s in the simulation i based on the estimation window length L. Let $\left(\hat{\boldsymbol{\mu}}^i, \hat{\Sigma}^i\right)$ be the sample mean and covariance matrix of simulation i. The mean and variance of portfolio $\tilde{\mathbf{w}}^i_{s,L}$ are given by:

$$\hat{\mu}^i_{s,L}\left(\tilde{\mathbf{w}}^i_{s,L}\right) = (\tilde{\mathbf{w}}^i_{s,L})'\hat{\boldsymbol{\mu}}^i \tag{30}$$

$$\hat{\sigma}^i_{s,L}\left(\tilde{\mathbf{w}}^i_{s,L}\right) = \sqrt{(\tilde{\mathbf{w}}^i_{s,L})'\hat{\Sigma}^i\tilde{\mathbf{w}}^i_{s,L}} \tag{31}$$

$$\mathrm{sr}(\tilde{\mathbf{w}}^i_{s,L}) = \frac{\hat{\mu}^i_{s,L}\left(\tilde{\mathbf{w}}^i_{s,L}\right)}{\hat{\sigma}^i_{s,L}\left(\tilde{\mathbf{w}}^i_{s,L}\right)} \tag{32}$$

The monthly Sharpe ratio of portfolio s is computed as the average cross 10000 simulations:

$$\mathrm{sr}_{s,L} = \frac{1}{10000}\sum_{i=1}^{10000}\mathrm{sr}(\tilde{\mathbf{w}}^i_{s,L}), \quad s = 1, 2, ..., 12; L = 120, 240, ..., 1200 \tag{33}$$

4.2. Results

In earlier sections, we showed analytically that the 1/N portfolio rule is unlikely to outperform mean-variance strategies when the CAPM does not hold. In this section, we investigate the quantitative implications of this claim. How large must the CAPM deviations be for the mean-variance strategies to outperform the 1/N rule? How many observations do we need to obtain sufficiently accurate estimates for mean-variance portfolios to outperform? We present the Sharpe ratios of the 12 portfolios under different simulation environments to answer these questions.

Table 2 reports the results for portfolio construction with 10 securities. Panel A confirms our analytical result that the 1/N rule consistently outperforms sample-based strategies when the CAPM holds. The 1/N rule has a Sharpe ratio of 13.45%, close to the highest possible Sharpe ratio of 14.43% attained by the *ex ante* tangency portfolio. This result is consistent with those from DeMiguel et al. (2009) and Tu and Zhou (2011).

Once we introduce mispricings relative to the CAPM, such that $\boldsymbol{\alpha} \neq \mathbf{0}$, the sample-based strategies start to outperform the 1/N rule. Even with small deviations from the CAPM drawn from Unif$[-1\%, 1\%]$, the MacKinlay-Por and CKZ strategies outperform 1/N when the estimation window is sufficiently long to obtain stable estimates. The MacKinlay-Por portfolio outperforms the 1/N rule with an estimation window of 960 months or longer, and the CKZ rule outperforms 1/N with an estimation window of at least 1080 months. The maximum possible Sharpe ratio captured by the *ex ante* tangency portfolio increases from 14.43% in Panel A to 14.72% in Panel B, which suggests that the mean-variance strategy exploiting mispricings can achieve a higher Sharpe ratio.

Sample-based mean-variance strategies are able to outperform the 1/N rule for two reasons. First, with constant mean and covariance matrix, a long enough estimation window gives mean-variance strategies reliable inputs. Second, mean-variance strategies seek to attain the highest Sharpe ratio by trading off expected returns and volatility. These strategies exploit mispricings for higher Sharpe ratios.

In Panels C, D, E, and F, the range of mispricings relative to the CAPM is set to Unif$[-i\%, i\%]$, for $i = 2, 3, 4, 5$. The Sharpe ratio of the *ex ante* tangency portfolio increases with the magnitude of mispricings. An increasing number of sample-based strategies obtain higher Sharpe ratios compared to the 1/N rule, with shorter estimation windows. In particular, with mispricings drawn from Unif$[-5\%, 5\%]$, the sample tangency portfolio outperforms the 1/N rule when the estimation window is as short as 240 months. Larger deviations from the CAPM allow the sample-based mean-variance strategies but not the 1/N rule to capture such deviations and improve Sharpe ratios.

Interestingly, not all sample-based strategies are able to beat the 1/N rule. Three portfolio rules are still dominated by the 1/N rule even when mispricings are large. The true minimum-variance ("min_True"), sample minimum-variance ("min_Sample") and the combination of the 1/N rule and minimum-variance ("ew-min") all have Sharpe ratios smaller than that of the 1/N portfolio. To sum up, we find the minimum-variance type of strategies can not outperform the 1/N rule, consistent with recent evidence from Wang et al. (2015).

Table 3 reports the results for portfolios with 25 securities. In Panel A, the Sharpe ratio of the *ex ante* tangency portfolio is 14.43%, the same as Panel A in Table 2. This observation arises in the absence of mispricings where the *ex ante* tangency portfolio is the market portfolio and the contribution of each individual security to the optimal portfolio is small (Treynor and Black, 1973). The Sharpe ratio of the 1/N rule increases from 13.45% for 10 securities to 13.99% for 25 securities, because a larger number of securities allows for better diversification. Most of the sample-based strategies have lower Sharpe ratios than the case with 10 securities, because estimation errors become more important with additional securities. In Panel A, no sample-based strategy outperforms the 1/N rule.

If the CAPM does not hold (Panels B through F), the Sharpe ratio of the *ex ante* tangency portfolio increases with the number of investable securities (N), especially when mispricings are large. With more securities to choose from and larger mispricings to exploit, the *ex ante* tangency portfolio can achieve increasingly higher Sharpe ratios. From Panel B to Panel F, the *ex ante* tangency portfolio has larger Sharpe ratios in Table 3 than in Table 2. Compared to the 1/N rule, the *ex ante* tangency portfolio has Sharpe ratios that are more sensitive to the number of securities (N).

Although the *ex ante* tangency portfolio has a Sharpe ratio increasing in N, this increase does not necessarily translate into sample-based mean-variance strategies. The difficulty is that a larger N also imply a greater number of parameters, which exacerbates the estimation error problem. As N grows, there is a tradeoff between accurately estimating the covariance matrix and exploiting mispricings. The former makes it more difficult to achieve a high Sharpe ratio, whereas the latter offers mean-variance strategies an advantage over the 1/N rule. In our simulations, for small deviations from the CAPM (less than 3%), estimation errors dominate when the estimation window is short. As N increases, the mispricing-exploitation effect dominates. Given a sufficiently large mispricing, sample-based strategies can outperform the 1/N rule.

Table 4 presents the results for 50 securities. The takeaways are qualitatively similar to those for 25 securities versus 10 securities: As the mispricing relative to the CAPM increases in magnitude, more and more sample-based mean-variance strategies are able to outperform the 1/N portfolio rule. Quantitative, the *ex ante* tangency and other mean-variance portfolios have much higher Sharpe ratios than the case with 25 or 10 securities as it exploits mispricings in more securities, and the increase in the Sharpe ratio for the *ex ante* tangency and other mean-variance portfolios from 25 to 50 securities in the presence of mispricing is much greater than the increase in Sharpe ratios for the 1/N rule. The overall robustness of our results are not affected by transaction costs, due to the huge differences between the Sharpe ratios of the mean-variance strategies and the 1/N rule (in many cases in Table 4, the Sharpe ratios for the sample-based mean-variance portfolios doubles the ones for the 1/N rule and approaches the one for the textitex ante efficient mean-variance strategy). For brevity we follow Tu and

Zhou (2011) and omit the results when we take transaction costs into account. Furthermore, transaction cost varies substantially in the types, locations and other characteristics of investors. As noted by Kirby and Ostdiek (2012), Transaction costs might be less of an issue for large institutional investors. Establishing or liquidating a portfolio position could plausibly cost as little as 5 bp for such investors. A natural implication, of course, is that the mean-variance strategies may work better for institutional investors.

5. Empirical Application

To illustrate our idea empirically, now we apply the 12 portfolio rules to real data sets used in DeMiguel et al. (2009) and Tu and Zhou (2011), as well as the Fama-French 25 size and value portfolios, the Fama-French 49 industry portfolios, the Fama and French (1992, 1993) factors, and the Carhart (1997) factors[4].

The key insight underlying our empirical application is simple. The data in DeMiguel et al. (2009) and Tu and Zhou (2011) range from July 1963 through November 2004, a relatively stable period excluding the late 2000s Global Financial Crisis (GFC). Since we hold the lack of pricing as the reason for the superior performance of the 1/N rule, we conjecture that the relative performance of the mean-variance ones will be improved if we extend the sample through the GFC.

We report monthly Sharpe ratios in Table 5 for portfolios formed on various asset returns from Ken French's website. The data is from July 1963 to December 2015. We use a 120-month window to estimate parameters for the sample-based mean-variance strategies. For the *ex ante* strategies, as the true mean and covariance matrix are unavailable for real data, we follow

[4] A detailed description can be found in DeMiguel et al. (2009) and at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

DeMiguel et al. (2009) and Tu and Zhou (2011) to approximate them using the estimates from the entire sample (in-sample rules). In Table 5, the first, third, fourth, and seventh columns have been considered in DeMiguel et al. (2009), and the fifth and sixth columns have been considered in Tu and Zhou (2011), but both over a shorter span of the sample.

Several interesting findings can be observed in Table 5. First, we find at least some of the mean-variance rules, including "Jorion", "Kan-Zhou, and "CKZ", are able to outperform the 1/N rule in all nine sets of assets considered, with the number of assets N ranging from N=3 to N=53. Second, as the number of assets N increases, both the 1/N rule and the mean-variance strategies have an increasing Sharpe ratio, but the mean-variance strategies show larger increases. For instance, when the number of assets N increases from 21 to 29, the Sharpe ratio of the 1/N rule increases about 10%, whereas the Sharpe ratios of the majority of the mean-variance rules nearly double. Third, the minimum-variance strategies cannot outperform the 1/N rule in most cases except for N=3 and N=4, when the assets are the Fama and French (1992, 1993) factors or Carhart (1997) factors. This result is probably due to the very small number of assets under consideration, combined with the fact that these factors have a relatively similar magnitude of the expected returns. In comparison, for the other sets of assets, we find negative Sharpe ratios for minimum-variance strategies.

To sum up, we confirm our three main findings using the actual rather than simulated data. Compared to DeMiguel et al. (2009) and Tu and Zhou (2011), we add the GFC and the post-crisis periods, when there are greater possibilities of mispricings relative to benchmark models. Whereas DeMiguel et al. (2009) find that the mean-variance strategies cannot beat the 1/N rule over the sample excluding the GFC, we identify the merits for a number of sample-based mean-variance strategies after taking into account the GFC and post-crisis periods. The performance of our mean-variance strategies become better if we use individual stocks instead of the well-diversified portfolios, which is consistent with our theoretical predictions.

6. Robustness

To illustrate our idea, we used simulated data in which the benchmark asset-pricing model is the CAPM. The reader may wonder if our results hold under more general settings: (i) Alternative data-generating processes (DGP), (ii) Alternative market error distributions such as Student-t and Normal Inverse Gaussian (NIG), (iii) Alternative time-varying volatility setting, (iv)Alternative portfolio performance measures to the Sharpe ratio, such as the Certainty-Equivalent Return (CER), and (v) Alternative simulation parameters such as the range of mispricings, the number of investable securities, and the length of estimation windows. We address these concerns in order.

We consider more complex models including the Fama and French (1992, 1993) and Carhart (1997) models, and find our results qualitatively unchanged. Until the empirical section, we have used the CAPM as our benchmark model: Mispricings were measured relative to the CAPM. Two issues arise: It is possible that the performance of some portfolio rules depends on the model specifications, and it is well-known that the CAPM is not able to capture the cross-sectional differences in average returns (Fama and French, 1992, 1993; MacKinlay, 1995). We consider the Fama and French (1992, 1993) and Carhart (1997) models in place of the CAPM as the true factor model. Of the 12 portfolio rules, only the *dm* and MacKinlay-Por portfolios are quantitatively affected, probably because their sample estimates depend on the underlying factor structure of returns.

We also examine non-normal return distributions. In our benchmark results, we used multivariate normal returns similar to the simulation settings in DeMiguel et al. (2009) and Tu and Zhou (2011). However, as asset returns deviate from normality, the covariance matrix becomes more difficult to estimate, making sample-based strategies less desirable. To address this problem, we hold the expected returns and volatility constant but increase the excess kurtosis to 4 by sampling market errors (mispricings) from Student-t and Normal Inverse Gaussian (NIG) distribution[5]. We repeat our simulations and find our results qualitatively unchanged. Table 6 and 7 report the Sharpe ratios when we sample market errors from an NIG distribution with zero mean and skewness, standard deviation = 0.2, and excess kurtosis = 4, when N = 25 and 50, respectively. When we increase the excess kurtosis to 11, it takes roughly an additional 120 months for sample-based mean-variance strategies to outperform the 1/N rule.

We also examine the case of time-varying idiosyncratic volatility. In our benchmark results, we used the assumption of constant volatility similar to the simulation settings in DeMiguel et al. (2009) and Tu and Zhou (2011). However, it is widely known that the asset idiosyncratic volatility is time-varying and subject to a Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) process. In the presence of time-varying volatility, the covariance matrix becomes more difficult to estimate, making sample-based strategies less desirable. To address this problem, we alternatively assume that the idiosyncratic volatility follows the GARCH(1,1) model with the parameters calibrated from the real data by Engle (2001). We repeat our simulations and find our results qualitatively unchanged. Table 8 and 9 report the Sharpe ratios when N = 10 and 25, respectively.

Using Certainty-Equivalent Return (CER) instead of Sharpe ratios do not change our results qualitatively. One advantage of the Sharpe ratio over the CER is that the CER depends on the risk-aversion coefficient whereas the Sharpe ratio does not. We compare our portfolio rules using CER, and find similar results. For instance, Table 10 and 11 report the Certainty-Equivalent Returns from 10 investable assets when the risk-aversion coefficient of 1 and 3, respectively. Tu and Zhou (2011) compare the CER under non-zero mispricing, and find the mean-variance strategies dominating the 1/N rule. However, it is less clear whether their result comes from the risk-aversion coefficients or mispricing or both. We argue that non-zero mispricing by itself is strong enough for mean-variance strategies to outperform the 1/N rule.

[5]As we find, the data used in DeMiguel et al. (2009) roughly have an excess kurtosis of 4.

We have also looked at the sensitivity of our results to the choice of simulation parameters, and found our results qualitatively unchanged under a broad set of parameters. To be specific, We have considered deviations from the benchmark model up to 50%, the number of months up to 6000, and the number of securities up to 1000.

7. Conclusion

In this paper, we investigate the relative merits of the 1/N rule and a broad set of mean-variance strategies through the lens of the mispricing relative to a cross-sectional asset pricing model. Using the CAPM for illustration, we derive an analytic expression to understand the Sharpe ratio of the 1/N rule. If the CAPM holds, with low idiosyncratic volatility relative to market volatility and a large number of assets N, the 1/N rule has a Sharpe ratio which approaches the maximum feasible Sharpe ratio, that of the *ex ante* tangency portfolio. This closed-form result explains the excellent performance of the 1/N rule documented in DeMiguel et al. (2009), without the need for simulation or empirical testing.

The excellent performance of the 1/N rule no longer holds when the underlying factor model does not capture the cross-sectional differences in average returns. The mean-variance rules make use of the mispricings, whereas the 1/N rule does not. When the number of securities N is large, although the sample-based mean-variance rules require more estimated parameters as inputs, they have more opportunities to exploit mispricings. This tradeoff does not guarantee the excellent performance of the 1/N rule – which effect is stronger comes down to an empirical question. Our simulations show that, given sufficiently large mispricings, an increase in N will cause the mean-variance rules to outperform the 1/N rule.

Our simulations also show that, as the magnitude of the mispricing grows, most sample-based mean-variance strategies are able to outperform the 1/N portfolio even at short horizons. This result implies that the benefit from increasing expected returns through mispricing exploitation can be large for mean-variance portfolios. The only exception is the minimum-variance type of portfolios. Because the minimum-variance portfolios are designed to achieve the lowest variance but not necessarily a high Sharpe ratio, they do not outperform the 1/N rule. We also confirm our simulation findings using actual data.

We illustrate that the performance of asset pricing models can be related to the effectiveness of mean-variance strategies. The better an asset pricing model is able to capture the variations in average returns in a set of investable assets, mispricings would be small, and the worse the sample-based mean-variance strategies will do relative to the 1/N rule. The sample-based mean-variance strategies exploit mispricings, but the mispricing itself may contain sampling variation. As a result, Bayesian methods that take into account the uncertainty associated with mispricing estimates may be an interesting area for future research.

Our paper has an important implication for the investors facing portfolio choice decisions. Many investors have found the mean-variance portfolio optimizations difficult to implement in practice because they are highly sensitive to inputs. In response, some investors shy away from mean-variance analysis completely. We find that the mean-variance analysis has its merits. In practice, often asset-pricing models are not able to capture all of the cross-sectional variations in average returns. Our work shows that under these circumstances, there is a role for the sample-based mean-variance strategies – they are likely to capture the mispricing and improve the portfolio performance.

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Table 1: **List of 12 Portfolio Rules**
This table lists all of the portfolio rules we consider. The abbreviations we use for the rules are listed on the right column. We divide portfolio rules into six categories. "Na" includes the 1/N rule. "Mean-Variance Tangency Portfolios" includes the tangency portfolio formed using both population and sample moments. "Global Minimum Variance Portfolios" includes the global minimum variance portfolio formed using population and sample moments. "Bayesian Approaches" includes two methods that incorporate Bayesian techniques. "Moment Restrictions" includes the MacKinlay-Por rule. "Portfolio Combinations" forms linear combinations of some of the above portfolio rules.

Model	Abbreviation
Na	
1. Equal-weight Portfolio with Rebalancing	ew, 1/N
Mean-Variance Tangency Portfolios	
2. *ex ante* Tangency Portfolio	True
3. Sample Tangency Portfolio	mv, Sample
Global Minimum Variance Portfolios	
4. True Global Minimum Variance Portfolio	min_True
5. Sample Global Minimum Variance Portfolio	min_Sample
Bayesian Approaches	
6. Jorion's (1986) Bayes-Stein Estimator	bs, Jorion
7. Por (2000), Por and Stambaugh (2000)	dm
Moment Restrictions	
8. MacKinlay and Por (2000)	mp, MacKinlay-Por
Portfolio Combinations	
9. Mixture of Minimum-Variance and 1/N	ew-min
10. Kan and Zhou's (2007) Three-Fund Model	mv-min, Kan-Zhou
11. Sample Tangency and 1/N	CML
12. Kan and Zhou's (2007) Three-Fund and 1/N	CKZ

Table 2: **Portfolios with 10 Investable Assets**

We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 10 assets. Estimation windows range from 120 to 1200 months. Returns are simulated from the market model: Equation (1), with $\boldsymbol{\beta}$ from Unif[0.5, 1.5], market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from $N(0, \sigma_i^2 I_N)$ where $\sigma_i \sim$ Unif[0.1, 0.3], all annualized. We draw $\boldsymbol{\alpha}$ from Unif[$-i$%, i%], where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
N=10					**Panel A: α=0**					
1/N	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45
True	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**
Sample	6.18	8.15	9.35	10.15	10.74	11.19	11.54	11.83	12.06	12.25
min_True	11.57	11.57	11.57	11.57	11.57	11.57	11.57	11.57	11.57	11.57
min_Sample	11.12	11.35	11.42	11.46	11.48	11.50	11.51	11.52	11.52	11.53
Jorion	7.95	9.97	10.97	11.57	11.95	12.24	12.45	12.62	12.76	12.87
dm	6.58	8.38	9.51	10.27	10.83	11.26	11.60	11.88	12.10	12.29
MacKinlay-Por	12.10	12.78	13.11	13.26	13.33	13.37	13.40	13.42	13.43	13.44
ew-min	12.36	12.02	11.88	11.81	11.76	11.73	11.71	11.69	11.68	11.67
Kan-Zhou	8.65	10.56	11.38	11.86	12.14	12.36	12.53	12.66	12.77	12.87
CML	9.88	11.39	11.97	12.22	12.35	12.47	12.55	12.63	12.72	12.81
CKZ	11.05	12.24	12.69	12.91	13.04	13.13	13.20	13.26	13.31	13.35
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45
True	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**
Sample	6.35	8.40	9.65	10.47	11.06	11.52	11.88	12.15	12.38	12.57
min_True	11.59	11.59	11.59	11.59	11.59	11.59	11.59	11.59	11.59	11.59
min_Sample	11.14	11.37	11.44	11.48	11.50	11.51	11.53	11.53	11.54	11.55
Jorion	8.07	10.18	11.23	11.82	12.21	12.51	12.73	12.89	13.03	13.14
dm	6.75	8.63	9.81	10.58	11.15	11.59	11.93	12.20	12.42	12.61
MacKinlay-Por	12.10	12.82	13.16	13.31	13.38	13.42	13.45	**13.47**	**13.48**	**13.49**
ew-min	12.37	12.03	11.90	11.82	11.78	11.75	11.73	11.71	11.70	11.69
Kan-Zhou	8.74	10.72	11.60	12.07	12.37	12.60	12.78	12.91	13.03	13.13
CML	9.82	11.41	12.00	12.28	12.42	12.56	12.67	12.76	12.86	12.95
CKZ	11.06	12.35	12.82	13.06	13.19	13.31	13.39	13.45	**13.51**	**13.55**
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46
True	**15.53**	**15.53**	**15.53**	**15.53**	**15.53**	**15.53**	**15.53**	**15.53**	**15.53**	**15.53**
Sample	6.99	9.20	10.50	11.33	11.95	12.40	12.75	13.03	13.27	13.46
min_True	11.55	11.55	11.55	11.55	11.55	11.55	11.55	11.55	11.55	11.55
min_Sample	11.10	11.33	11.40	11.44	11.46	11.48	11.49	11.49	11.50	11.51
Jorion	8.60	10.80	11.88	12.50	12.92	13.22	13.46	**13.64**	**13.79**	**13.92**
dm	7.37	9.41	10.64	11.43	12.02	12.46	12.80	13.07	13.30	13.48
MacKinlay-Por	12.22	13.00	13.33	**13.48**	**13.55**	**13.59**	**13.62**	**13.64**	**13.65**	**13.66**
ew-min	12.34	12.00	11.86	11.79	11.75	11.72	11.69	11.67	11.66	11.65
Kan-Zhou	9.18	11.24	12.16	12.67	13.00	13.26	**13.47**	**13.63**	**13.77**	**13.89**
CML	10.04	11.65	12.27	12.58	12.80	12.98	**13.18**	**13.34**	**13.49**	**13.63**
CKZ	11.39	12.71	13.23	**13.50**	**13.68**	**13.82**	**13.94**	**14.04**	**14.12**	**14.20**

Table 2 (*Continued*)

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel D: $\alpha \sim$ Unif [-3%, 3%]					
1/N	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45	13.45
True	**16.77**	**16.77**	**16.77**	**16.77**	**16.77**	**16.77**	**16.77**	**16.77**	**16.77**	**16.77**
Sample	8.14	10.51	11.84	12.73	13.32	**13.76**	**14.10**	**14.37**	**14.60**	**14.78**
min_True	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56
min_Sample	11.13	11.34	11.42	11.45	11.47	11.49	11.50	11.50	11.51	11.51
Jorion	9.58	11.87	12.99	**13.66**	**14.09**	**14.40**	**14.64**	**14.83**	**14.99**	**15.12**
dm	8.49	10.70	11.96	12.81	13.38	**13.81**	**14.14**	**14.41**	**14.63**	**14.80**
MacKinlay-Por	12.48	13.28	**13.61**	**13.75**	**13.83**	**13.86**	**13.88**	**13.90**	**13.90**	**13.91**
ew-min	12.36	12.01	11.88	11.80	11.76	11.73	11.70	11.68	11.67	11.66
Kan-Zhou	10.00	12.16	13.15	**13.72**	**14.11**	**14.39**	**14.61**	**14.79**	**14.95**	**15.07**
CML	10.54	12.17	**12.87**	**13.31**	**13.65**	**13.95**	**14.20**	**14.41**	**14.61**	**14.78**
CKZ	12.03	13.41	**13.99**	**14.34**	**14.58**	**14.77**	**14.93**	**15.06**	**15.18**	**15.27**
					Panel E: $\alpha \sim$ Unif [-4%, 4%]					
1/N	13.44	13.44	13.44	13.44	13.44	13.44	13.44	13.44	13.44	13.44
True	**18.30**	**18.30**	**18.30**	**18.30**	**18.30**	**18.30**	**18.30**	**18.30**	**18.30**	**18.30**
Sample	9.45	12.04	**13.46**	**14.34**	**14.96**	**15.41**	**15.76**	**16.03**	**16.24**	**16.42**
min_True	11.57	11.57	11.57	11.57	11.57	11.57	11.57	11.57	11.57	11.57
min_Sample	11.13	11.36	11.43	11.47	11.48	11.50	11.51	11.52	11.52	11.52
Jorion	10.72	13.18	**14.38**	**15.07**	**15.54**	**15.88**	**16.14**	**16.35**	**16.51**	**16.65**
dm	9.77	12.21	**13.56**	**14.41**	**15.02**	**15.45**	**15.79**	**16.06**	**16.27**	**16.43**
MacKinlay-Por	12.71	**13.63**	**13.97**	**14.11**	**14.20**	**14.23**	**14.25**	**14.26**	**14.27**	**14.27**
ew-min	12.36	12.02	11.88	11.81	11.76	11.73	11.71	11.69	11.68	11.67
Kan-Zhou	10.99	13.35	**14.45**	**15.08**	**15.51**	**15.83**	**16.09**	**16.29**	**16.46**	**16.60**
CML	11.10	12.97	**13.91**	**14.50**	**14.96**	**15.34**	**15.64**	**15.90**	**16.12**	**16.32**
CKZ	12.81	**14.37**	**15.09**	**15.53**	**15.85**	**16.10**	**16.30**	**16.47**	**16.60**	**16.72**
					Panel F: $\alpha \sim$ Unif [-5%, 5%]					
1/N	13.44	13.44	13.44	13.44	13.44	13.44	13.44	13.44	13.44	13.44
True	**20.11**	**20.11**	**20.11**	**20.11**	**20.11**	**20.11**	**20.11**	**20.11**	**20.11**	**20.11**
Sample	11.14	**13.93**	**15.40**	**16.30**	**16.91**	**17.37**	**17.69**	**17.95**	**18.15**	**18.33**
min_True	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56
min_Sample	11.12	11.32	11.39	11.43	11.45	11.48	11.49	11.50	11.50	11.51
Jorion	12.19	**14.82**	**16.09**	**16.82**	**17.32**	**17.69**	**17.95**	**18.16**	**18.33**	**18.48**
dm	11.42	**14.08**	**15.50**	**16.36**	**16.96**	**17.40**	**17.72**	**17.97**	**18.17**	**18.34**
MacKinlay-Por	13.18	**14.09**	**14.47**	**14.63**	**14.69**	**14.72**	**14.73**	**14.74**	**14.74**	**14.75**
ew-min	12.35	11.99	11.85	11.78	11.74	11.71	11.69	11.68	11.66	11.65
Kan-Zhou	12.29	**14.86**	**16.07**	**16.77**	**17.25**	**17.62**	**17.89**	**18.10**	**18.28**	**18.43**
CML	12.02	**14.20**	**15.36**	**16.13**	**16.68**	**17.14**	**17.50**	**17.79**	**18.02**	**18.22**
CKZ	**14.79**	**15.95**	**16.56**	**16.94**	**17.23**	**17.46**	**17.63**	**17.78**	**17.90**	**18.01**

Table 3: **Portfolios with 25 Investable Assets**
We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 25 assets. Estimation windows range from 120 to 1200 months. Returns are simulated from the market model: Equation (1), with $\boldsymbol{\beta}$ from Unif[0.5, 1.5], market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from $N(0, \sigma_i^2 I_N)$ where $\sigma_i \sim$ Unif[0.1, 0.3], all annualized. We draw $\boldsymbol{\alpha}$ from Unif[$-i$%, i%], where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel A: α=0					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**
Sample	3.86	5.57	6.69	7.52	8.17	8.70	9.15	9.52	9.84	10.13
min_True	9.32	9.32	9.32	9.32	9.32	9.32	9.32	9.32	9.32	9.32
min_Sample	8.33	8.84	9.01	9.09	9.14	9.17	9.19	9.21	9.22	9.23
Jorion	4.87	6.86	8.02	8.82	9.42	9.87	10.24	10.54	10.79	11.01
dm	4.57	6.05	7.04	7.80	8.40	8.89	9.31	9.66	9.97	10.25
MacKinlay-Por	12.98	13.54	13.78	13.88	13.92	13.94	13.96	13.97	13.97	13.98
ew-min	11.60	10.56	10.18	9.98	9.85	9.76	9.70	9.65	9.62	9.59
Kan-Zhou	5.90	7.95	8.96	9.60	10.03	10.35	10.61	10.83	11.02	11.19
CML	10.30	11.80	12.42	12.73	12.90	13.01	13.09	13.16	13.21	13.25
CKZ	10.66	11.55	12.00	12.25	12.41	12.52	12.59	12.66	12.71	12.75
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**15.17**	**15.17**	**15.17**	**15.17**	**15.17**	**15.17**	**15.17**	**15.17**	**15.17**	**15.17**
Sample	4.26	6.12	7.31	8.19	8.87	9.42	9.88	10.28	10.61	10.90
min_True	9.32	9.32	9.32	9.32	9.32	9.32	9.32	9.32	9.32	9.32
min_Sample	8.33	8.85	9.01	9.09	9.13	9.16	9.19	9.20	9.22	9.23
Jorion	5.27	7.37	8.58	9.41	10.01	10.49	10.87	11.19	11.46	11.69
dm	4.96	6.59	7.65	8.46	9.09	9.60	10.03	10.41	10.73	11.01
MacKinlay-Por	13.07	13.61	13.85	13.94	13.98	**14.01**	**14.02**	**14.03**	**14.04**	**14.04**
ew-min	11.59	10.57	10.17	9.97	9.84	9.76	9.70	9.65	9.61	9.58
Kan-Zhou	6.22	8.36	9.40	10.06	10.52	10.87	11.16	11.41	11.62	11.80
CML	10.31	11.88	12.50	12.81	12.98	13.10	13.19	13.27	13.34	13.40
CKZ	10.80	11.74	12.18	12.46	12.63	12.76	12.86	12.95	13.03	13.10
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**17.17**	**17.17**	**17.17**	**17.17**	**17.17**	**17.17**	**17.17**	**17.17**	**17.17**	**17.17**
Sample	5.41	7.65	9.09	10.11	10.88	11.49	11.98	12.40	12.75	13.06
min_True	9.31	9.31	9.31	9.31	9.31	9.31	9.31	9.31	9.31	9.31
min_Sample	8.32	8.84	9.00	9.08	9.12	9.15	9.17	9.19	9.20	9.21
Jorion	6.28	8.73	10.16	11.11	11.81	12.34	12.74	13.09	13.38	13.64
dm	6.07	8.07	9.40	10.34	11.07	11.65	12.11	12.51	12.85	13.15
MacKinlay-Por	13.19	13.83	**14.08**	**14.15**	**14.19**	**14.21**	**14.22**	**14.23**	**14.23**	**14.24**
ew-min	11.59	10.56	10.17	9.96	9.83	9.75	9.68	9.64	9.60	9.57
Kan-Zhou	6.89	9.40	10.68	11.49	12.08	12.51	12.87	13.17	13.42	13.65
CML	10.56	12.26	12.97	13.35	13.64	13.85	**14.04**	**14.22**	**14.37**	**14.51**
CKZ	11.28	12.38	13.00	13.38	13.65	13.87	**14.05**	**14.21**	**14.35**	**14.48**

Table 3 (*Continued*)

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel D: $\alpha \sim$ Unif [-3%, 3%]					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**19.96**	**19.96**	**19.96**	**19.96**	**19.96**	**19.96**	**19.96**	**19.96**	**19.96**	**19.96**
Sample	7.25	10.04	11.71	12.87	13.74	**14.41**	**14.94**	**15.38**	**15.75**	**16.06**
min_True	9.36	9.36	9.36	9.36	9.36	9.36	9.36	9.36	9.36	9.36
min_Sample	8.35	8.87	9.04	9.12	9.17	9.20	9.22	9.24	9.25	9.26
Jorion	8.01	10.94	12.56	13.64	**14.42**	**15.02**	**15.48**	**15.87**	**16.18**	**16.45**
dm	7.86	10.41	11.97	13.07	13.90	**14.54**	**15.04**	**15.47**	**15.82**	**16.12**
MacKinlay-Por	13.47	**14.20**	**14.41**	**14.49**	**14.52**	**14.54**	**14.54**	**14.55**	**14.55**	**14.55**
ew-min	11.60	10.58	10.20	10.00	9.88	9.79	9.73	9.68	9.65	9.62
Kan-Zhou	8.24	11.25	12.79	13.78	**14.50**	**15.05**	**15.48**	**15.85**	**16.15**	**16.42**
CML	11.22	13.18	**14.16**	**14.80**	**15.29**	**15.70**	**16.04**	**16.33**	**16.58**	**16.80**
CKZ	12.17	13.68	**14.54**	**15.12**	**15.56**	**15.93**	**16.22**	**16.48**	**16.70**	**16.89**
					Panel E: $\alpha \sim$ Unif [-4%, 4%]					
1/N	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98
True	**23.20**	**23.20**	**23.20**	**23.20**	**23.20**	**23.20**	**23.20**	**23.20**	**23.20**	**23.20**
Sample	9.50	12.97	**14.92**	**16.19**	**17.13**	**17.84**	**18.40**	**18.86**	**19.23**	**19.54**
min_True	9.31	9.31	9.31	9.31	9.31	9.31	9.31	9.31	9.31	9.31
min_Sample	8.30	8.84	9.01	9.09	9.12	9.15	9.18	9.20	9.21	9.22
Jorion	10.12	13.67	**15.55**	**16.75**	**17.62**	**18.25**	**18.76**	**19.17**	**19.50**	**19.78**
dm	10.06	13.29	**15.13**	**16.35**	**17.25**	**17.93**	**18.48**	**18.93**	**19.28**	**19.59**
MacKinlay-Por	13.87	**14.67**	**14.90**	**14.96**	**14.99**	**14.99**	**14.99**	**15.00**	**15.00**	**14.99**
ew-min	11.56	10.55	10.17	9.97	9.83	9.75	9.69	9.64	9.61	9.58
Kan-Zhou	10.00	13.71	**15.56**	**16.74**	**17.58**	**18.20**	**18.69**	**19.11**	**19.44**	**19.73**
CML	12.10	**14.81**	**16.22**	**17.21**	**17.93**	**18.48**	**18.93**	**19.32**	**19.64**	**19.90**
CKZ	13.41	**15.61**	**16.80**	**17.64**	**18.27**	**18.75**	**19.14**	**19.48**	**19.76**	**19.99**
					Panel F: $\alpha \sim$ Unif [-5%, 5%]					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**26.90**	**26.90**	**26.90**	**26.90**	**26.90**	**26.90**	**26.90**	**26.90**	**26.90**	**26.90**
Sample	12.46	**16.52**	**18.71**	**20.10**	**21.07**	**21.80**	**22.37**	**22.82**	**23.18**	**23.48**
min_True	9.34	9.34	9.34	9.34	9.34	9.34	9.34	9.34	9.34	9.34
min_Sample	8.30	8.83	9.00	9.09	9.14	9.17	9.20	9.22	9.23	9.24
Jorion	12.92	**17.01**	**19.15**	**20.46**	**21.38**	**22.06**	**22.58**	**23.00**	**23.34**	**23.62**
dm	12.95	**16.78**	**18.88**	**20.22**	**21.16**	**21.88**	**22.43**	**22.87**	**23.22**	**23.52**
MacKinlay-Por	**14.41**	**15.28**	**15.54**	**15.59**	**15.60**	**15.60**	**15.59**	**15.59**	**15.59**	**15.58**
ew-min	11.57	10.55	10.17	9.97	9.85	9.76	9.71	9.66	9.63	9.60
Kan-Zhou	12.34	**16.78**	**18.98**	**20.33**	**21.27**	**21.97**	**22.51**	**22.94**	**23.28**	**23.58**
CML	13.53	**17.20**	**19.17**	**20.47**	**21.39**	**22.07**	**22.61**	**23.04**	**23.38**	**23.67**
CKZ	**15.20**	**18.19**	**19.84**	**20.92**	**21.69**	**22.29**	**22.77**	**23.15**	**23.46**	**23.73**

Table 4: **Portfolios with 50 Investable Assets**
We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 50 assets. Estimation windows range from 120 to 1200 months. Returns are simulated from the market model: Equation (1), with $\boldsymbol{\beta}$ from Unif[0.5, 1.5], market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from $N(0, \sigma_i^2 I_N)$ where $\sigma_i \sim$ Unif[0.1, 0.3], all annualized. We draw $\boldsymbol{\alpha}$ from Unif[$-i$%, i%], where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
Panel A: α=0										
1/N	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20
True	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**
Sample	2.44	3.86	4.84	5.57	6.18	6.69	7.12	7.51	7.86	8.17
min_True	7.43	7.43	7.43	7.43	7.43	7.43	7.43	7.43	7.43	7.43
min_Sample	5.70	6.63	6.90	7.04	7.12	7.18	7.21	7.24	7.26	7.28
Jorion	2.88	4.56	5.65	6.43	7.06	7.56	7.99	8.36	8.68	8.96
dm	3.41	4.58	5.41	6.05	6.59	7.04	7.44	7.80	8.11	8.40
MacKinlay-Por	13.46	13.92	14.07	14.13	14.17	14.18	14.19	14.19	14.20	14.20
ew-min	11.95	9.76	8.98	8.59	8.36	8.20	8.09	8.00	7.94	7.89
Kan-Zhou	3.40	5.55	6.67	7.38	7.90	8.30	8.63	8.91	9.14	9.35
CML	10.74	12.17	12.76	13.06	13.24	13.35	13.43	13.47	13.50	13.53
CKZ	11.63	11.59	11.84	12.00	12.14	12.22	12.29	12.35	12.39	12.43
Panel B: $\alpha \sim$ Unif [-1%, 1%]										
1/N	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20
True	**15.91**	**15.91**	**15.91**	**15.91**	**15.91**	**15.91**	**15.91**	**15.91**	**15.91**	**15.91**
Sample	2.89	4.66	5.79	6.67	7.36	7.94	8.43	8.86	9.24	9.58
min_True	7.40	7.40	7.40	7.40	7.40	7.40	7.40	7.40	7.40	7.40
min_Sample	5.69	6.61	6.88	7.02	7.10	7.15	7.19	7.21	7.24	7.25
Jorion	3.30	5.32	6.54	7.46	8.15	8.73	9.20	9.60	9.95	10.27
dm	3.83	5.36	6.34	7.13	7.74	8.27	8.72	9.12	9.47	9.79
MacKinlay-Por	13.51	13.96	14.15	**14.21**	**14.24**	**14.25**	**14.26**	**14.26**	**14.26**	**14.27**
ew-min	11.93	9.73	8.95	8.56	8.33	8.17	8.06	7.98	7.91	7.86
Kan-Zhou	3.63	6.08	7.32	8.15	8.75	9.23	9.61	9.95	10.23	10.50
CML	10.70	12.21	12.83	13.13	13.31	13.44	13.52	13.60	13.66	13.71
CKZ	11.69	11.81	12.10	12.33	12.49	12.63	12.72	12.80	12.88	12.94
Panel C: $\alpha \sim$ Unif [-2%, 2%]										
1/N	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20
True	**19.54**	**19.54**	**19.54**	**19.54**	**19.54**	**19.54**	**19.54**	**19.54**	**19.54**	**19.54**
Sample	4.45	6.99	8.55	9.69	10.57	11.30	11.91	12.43	12.87	13.27
min_True	7.41	7.41	7.41	7.41	7.41	7.41	7.41	7.41	7.41	7.41
min_Sample	5.69	6.63	6.91	7.04	7.11	7.17	7.20	7.23	7.25	7.27
Jorion	4.81	7.55	9.17	10.32	11.18	11.88	12.47	12.96	13.37	13.74
dm	5.37	7.64	9.04	10.08	10.90	11.57	12.14	12.64	13.05	13.43
MacKinlay-Por	13.76	**14.22**	**14.37**	**14.43**	**14.45**	**14.46**	**14.47**	**14.47**	**14.47**	**14.47**
ew-min	11.94	9.76	8.98	8.59	8.35	8.19	8.08	8.00	7.93	7.88
Kan-Zhou	4.60	7.78	9.46	10.58	11.38	12.04	12.58	13.04	13.43	13.78
CML	10.95	12.72	13.49	13.98	**14.33**	**14.64**	**14.92**	**15.14**	**15.36**	**15.55**
CKZ	12.12	12.68	13.24	13.68	14.01	**14.31**	**14.58**	**14.80**	**15.01**	**15.20**

Table 4 (*Continued*)

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel D: $\alpha \sim$ Unif [-3%, 3%]					
1/N	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20	14.20
True	**24.44**	**24.44**	**24.44**	**24.44**	**24.44**	**24.44**	**24.44**	**24.44**	**24.44**	**24.44**
Sample	6.85	10.46	12.61	14.11	**15.24**	**16.15**	**16.87**	**17.47**	**17.98**	**18.41**
min_True	7.40	7.40	7.40	7.40	7.40	7.40	7.40	7.40	7.40	7.40
min_Sample	5.63	6.56	6.86	7.00	7.09	7.15	7.18	7.21	7.23	7.25
Jorion	7.11	10.87	13.07	**14.56**	**15.67**	**16.54**	**17.24**	**17.81**	**18.28**	**18.69**
dm	7.71	11.03	13.03	**14.44**	**15.50**	**16.36**	**17.05**	**17.63**	**18.11**	**18.53**
MacKinlay-Por	14.05	**14.60**	**14.75**	**14.80**	**14.81**	**14.81**	**14.81**	**14.81**	**14.81**	**14.81**
ew-min	11.88	9.68	8.92	8.53	8.31	8.16	8.06	7.97	7.91	7.86
Kan-Zhou	6.04	10.47	12.92	**14.50**	**15.63**	**16.50**	**17.19**	**17.76**	**18.24**	**18.65**
CML	11.29	13.89	**15.40**	**16.45**	**17.22**	**17.85**	**18.37**	**18.81**	**19.18**	**19.50**
CKZ	12.85	**14.43**	**15.66**	**16.57**	**17.26**	**17.84**	**18.32**	**18.73**	**19.08**	**19.39**
					Panel E: $\alpha \sim$ Unif [-4%, 4%]					
1/N	14.19	14.19	14.19	14.19	14.19	14.19	14.19	14.19	14.19	14.19
True	**29.74**	**29.74**	**29.74**	**29.74**	**29.74**	**29.74**	**29.74**	**29.74**	**29.74**	**29.74**
Sample	9.81	**14.76**	**17.50**	**19.29**	**20.59**	**21.60**	**22.40**	**23.05**	**23.59**	**24.05**
min_True	7.41	7.41	7.41	7.41	7.41	7.41	7.41	7.41	7.41	7.41
min_Sample	5.67	6.60	6.89	7.02	7.11	7.16	7.19	7.22	7.24	7.26
Jorion	9.99	**15.06**	**17.81**	**19.58**	**20.86**	**21.84**	**22.62**	**23.24**	**23.76**	**24.21**
dm	10.61	**15.25**	**17.83**	**19.54**	**20.79**	**21.76**	**22.53**	**23.16**	**23.68**	**24.13**
MacKinlay-Por	**14.54**	**15.16**	**15.27**	**15.32**	**15.31**	**15.31**	**15.30**	**15.30**	**15.29**	**15.29**
ew-min	11.93	9.73	8.96	8.57	8.34	8.18	8.07	7.98	7.92	7.87
Kan-Zhou	8.14	**14.32**	**17.40**	**19.34**	**20.69**	**21.72**	**22.52**	**23.17**	**23.70**	**24.16**
CML	12.08	**16.34**	**18.74**	**20.37**	**21.54**	**22.43**	**23.13**	**23.71**	**24.18**	**24.58**
CKZ	14.04	**17.27**	**19.29**	**20.67**	**21.69**	**22.51**	**23.16**	**23.70**	**24.15**	**24.55**
					Panel F: $\alpha \sim$ Unif [-5%, 5%]					
1/N	14.21	14.21	14.21	14.21	14.21	14.21	14.21	14.21	14.21	14.21
True	**35.35**	**35.35**	**35.35**	**35.35**	**35.35**	**35.35**	**35.35**	**35.35**	**35.35**	**35.35**
Sample	13.45	**19.67**	**22.84**	**24.91**	**26.35**	**27.40**	**28.26**	**28.94**	**29.49**	**29.95**
min_True	7.49	7.49	7.49	7.49	7.49	7.49	7.49	7.49	7.49	7.49
min_Sample	5.70	6.67	6.97	7.11	7.17	7.22	7.26	7.30	7.32	7.33
Jorion	13.52	**19.87**	**23.04**	**25.10**	**26.51**	**27.54**	**28.38**	**29.05**	**29.59**	**30.03**
dm	14.18	**20.08**	**23.12**	**25.11**	**26.50**	**27.52**	**28.36**	**29.02**	**29.56**	**30.00**
MacKinlay-Por	**15.14**	**15.82**	**15.97**	**15.98**	**15.96**	**15.94**	**15.93**	**15.91**	**15.90**	**15.89**
ew-min	11.96	9.80	9.03	8.65	8.41	8.24	8.13	8.06	8.00	7.94
Kan-Zhou	10.86	**18.88**	**22.55**	**24.81**	**26.33**	**27.42**	**28.28**	**28.97**	**29.53**	**29.99**
CML	13.32	**19.82**	**23.19**	**25.32**	**26.74**	**27.79**	**28.61**	**29.25**	**29.78**	**30.21**
CKZ	**15.83**	**21.04**	**23.80**	**25.64**	**26.92**	**27.87**	**28.65**	**29.27**	**29.78**	**30.20**

Table 5: **Portfolios Formed on Fama-French Factors and Industry Portfolios**

We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios forms on combinations of Fama-French factors as well as factor-based and industry portfolios, July 1963 through December 2015. N=3 corresponds to portfolios formed using the Fama and French (1992, 1993) factors. N=4 uses the Carhart (1997) factors. N=21 uses 20 portfolios formed on size and value ratios, plus the market factor. N=23 uses 20 size and value portfolios plus the Fama and French (1992, 1993) factors. N=28 uses the 25 size and book-to-market portfolios, plus the Fama and French (1992, 1993) factors. N=52 uses 49 industry portfolios plus the Fama and French (1992, 1993) factors. N=24 uses 20 size and value portfolios plus the Carhart (1997) factors. N=29 uses the 25 size and value portfolios plus the Carhart (1997) factors. N=53 uses the 49 industry portfolios plus the Carhart (1997) factors. Bold font indicates a superior performance relative to the naïve 1/N rule.

Rules\Assets	N=3	N=4	N=21	N=23	N=28	N=52	N=24	N=29	N=53
1/N	18.13	26.79	20.74	20.94	21.39	14.33	21.64	22.00	14.60
True (in-sample)	**20.47**	**30.09**	**56.05**	**93.90**	**98.69**	**81.60**	**96.48**	**100.85**	**92.61**
Sample	16.58	25.41	**40.34**	**69.93**	**72.16**	**16.50**	**71.84**	**73.61**	**24.28**
min_True (in-sample)	**19.57**	**28.23**	19.23	-71.07	-12.07	-15.28	-69.06	-11.99	-17.40
min_Sample	**19.02**	26.69	18.44	-59.98	-15.89	-3.36	-59.71	-15.35	-5.41
Jorion	**18.57**	**27.01**	**39.14**	**70.07**	**72.06**	**16.15**	**71.59**	**73.51**	**23.58**
dm	16.57	25.42	**39.36**	**68.15**	**72.20**	**16.90**	**72.03**	**73.78**	**24.66**
MacKinlay-Por	15.00	22.58	**22.06**	**22.14**	**22.36**	13.16	**22.63**	**22.79**	13.30
ew-min	**19.24**	**27.36**	20.56	-59.43	-15.47	-1.73	-59.12	-14.89	-3.60
Kan-Zhou	**18.54**	**27.07**	**37.56**	**69.83**	**72.00**	**15.13**	**71.17**	**73.45**	**21.96**
CML	15.04	18.89	**36.87**	**66.51**	**71.80**	13.89	**64.14**	**73.39**	**20.64**
CKZ	**18.75**	**27.71**	**38.68**	**69.99**	**71.49**	**18.34**	**71.89**	**73.37**	**24.80**

Table 6: **Portfolios with 25 Investable Assets (NIG errors)**
We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 25 assets. Returns are simulated from Equation (1), with $\boldsymbol{\beta}$ from Unif$[0.5, 1.5]$, market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from an NIG distribution with zero mean and skewness, standard deviation = 0.2, and excess kutosis = 4, all annualized. We draw $\boldsymbol{\alpha}$ from Unif$[-i\%, i\%]$, where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel A: α=0					
1/N	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98
True	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**
Sample	3.71	5.48	6.62	7.45	8.11	8.65	9.11	9.48	9.82	10.11
min_True	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85
min_Sample	7.21	7.98	8.25	8.39	8.48	8.54	8.59	8.62	8.64	8.66
Jorion	4.48	6.53	7.73	8.54	9.14	9.62	10.01	10.32	10.60	10.82
dm	4.43	5.97	6.98	7.74	8.35	8.84	9.27	9.63	9.95	10.22
MacKinlay-Por	12.83	13.52	13.78	13.87	13.92	13.94	13.96	13.97	13.97	13.98
ew-min	10.67	9.73	9.43	9.28	9.19	9.13	9.09	9.06	9.04	9.02
Kan-Zhou	5.15	7.35	8.44	9.12	9.60	9.96	10.26	10.50	10.71	10.90
CML	10.08	11.69	12.38	12.71	12.87	12.98	13.06	13.11	13.17	13.21
CKZ	10.25	11.23	11.75	12.04	12.21	12.34	12.43	12.50	12.57	12.62
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98
True	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**
Sample	4.16	6.02	7.23	8.13	8.82	9.40	9.87	10.26	10.60	10.90
min_True	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84
min_Sample	7.23	7.97	8.24	8.38	8.47	8.52	8.57	8.60	8.62	8.64
Jorion	4.90	7.01	8.26	9.12	9.76	10.27	10.67	11.00	11.28	11.52
dm	4.85	6.48	7.57	8.40	9.04	9.58	10.02	10.40	10.72	11.00
MacKinlay-Por	12.87	13.57	13.83	13.94	**13.99**	**14.01**	**14.02**	**14.03**	**14.04**	**14.04**
ew-min	10.69	9.72	9.42	9.27	9.17	9.11	9.07	9.04	9.02	9.00
Kan-Zhou	5.47	7.69	8.85	9.57	10.09	10.50	10.82	11.08	11.32	11.53
CML	10.08	11.75	12.43	12.75	12.96	13.08	13.17	13.24	13.31	13.37
CKZ	10.39	11.43	11.97	12.27	12.48	12.62	12.73	12.82	12.91	12.99
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**17.23**	**17.23**	**17.23**	**17.23**	**17.23**	**17.23**	**17.23**	**17.23**	**17.23**	**17.23**
Sample	5.33	7.66	9.08	10.10	10.88	11.51	12.01	12.44	12.80	13.11
min_True	8.86	8.86	8.86	8.86	8.86	8.86	8.86	8.86	8.86	8.86
min_Sample	7.24	8.00	8.27	8.41	8.49	8.55	8.59	8.62	8.65	8.67
Jorion	5.97	8.50	9.92	10.87	11.58	12.14	12.58	12.95	13.26	13.51
dm	5.98	8.08	9.39	10.33	11.06	11.66	12.14	12.55	12.90	13.19
MacKinlay-Por	13.07	13.78	**14.06**	**14.15**	**14.20**	**14.22**	**14.23**	**14.23**	**14.24**	**14.24**
ew-min	10.69	9.76	9.45	9.29	9.20	9.14	9.10	9.06	9.04	9.02
Kan-Zhou	6.23	8.87	10.16	11.02	11.65	12.14	12.54	12.88	13.18	13.44
CML	10.35	12.18	12.89	13.28	13.56	13.77	13.96	**14.13**	**14.30**	**14.45**
CKZ	10.91	12.13	12.76	13.15	13.44	13.68	13.88	**14.05**	**14.22**	**14.35**

Table 7: **Portfolios with 50 Investable Assets (NIG errors)**
We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 50 assets. Returns are simulated from Equation (1), with $\boldsymbol{\beta}$ from Unif$[0.5, 1.5]$, market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from an NIG distribution with zero mean and skewness, standard deviation = 0.2, and excess kutosis = 4, all annualized. We draw $\boldsymbol{\alpha}$ from Unif$[-i\%, i\%]$, where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel A: α=0					
1/N	13.97	13.97	13.97	13.97	13.97	13.97	13.97	13.97	13.97	13.97
True	**27.52**	**27.52**	**27.52**	**27.52**	**27.52**	**27.52**	**27.52**	**27.52**	**27.52**	**27.52**
Sample	13.04	**17.03**	**19.23**	**20.67**	**21.67**	**22.41**	**22.98**	**23.43**	**23.80**	**24.11**
min_True	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84
min_Sample	7.21	7.97	8.24	8.37	8.46	8.52	8.56	8.59	8.62	8.64
Jorion	13.28	**17.29**	**19.44**	**20.83**	**21.80**	**22.50**	**23.06**	**23.50**	**23.86**	**24.16**
dm	13.45	**17.26**	**19.38**	**20.77**	**21.75**	**22.46**	**23.03**	**23.47**	**23.84**	**24.14**
MacKinlay-Por	**14.26**	**15.21**	**15.48**	**15.56**	**15.58**	**15.58**	**15.59**	**15.58**	**15.57**	**15.57**
ew-min	10.67	9.73	9.42	9.26	9.17	9.11	9.07	9.04	9.01	8.99
Kan-Zhou	12.30	**16.73**	**19.09**	**20.61**	**21.64**	**22.40**	**22.98**	**23.44**	**23.82**	**24.13**
CML	13.72	**17.24**	**19.34**	**20.79**	**21.80**	**22.55**	**23.12**	**23.56**	**23.92**	**24.23**
CKZ	**15.19**	**18.15**	**19.93**	**21.15**	**22.04**	**22.69**	**23.21**	**23.63**	**23.97**	**24.26**
					Panel B: $\alpha\sim$ Unif [-1%, 1%]					
1/N	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98
True	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**
Sample	4.16	6.02	7.23	8.13	8.82	9.40	9.87	10.26	10.60	10.90
min_True	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84
min_Sample	7.23	7.97	8.24	8.38	8.47	8.52	8.57	8.60	8.62	8.64
Jorion	4.90	7.01	8.26	9.12	9.76	10.27	10.67	11.00	11.28	11.52
dm	4.85	6.48	7.57	8.40	9.04	9.58	10.02	10.40	10.72	11.00
MacKinlay-Por	12.87	13.57	13.83	13.94	**13.99**	**14.01**	**14.02**	**14.03**	**14.04**	**14.04**
ew-min	10.69	9.72	9.42	9.27	9.17	9.11	9.07	9.04	9.02	9.00
Kan-Zhou	5.47	7.69	8.85	9.57	10.09	10.50	10.82	11.08	11.32	11.53
CML	10.08	11.75	12.43	12.75	12.96	13.08	13.17	13.24	13.31	13.37
CKZ	10.39	11.43	11.97	12.27	12.48	12.62	12.73	12.82	12.91	12.99
					Panel C: $\alpha\sim$ Unif [-2%, 2%]					
1/N	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98
True	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**
Sample	4.16	6.02	7.23	8.13	8.82	9.40	9.87	10.26	10.60	10.90
min_True	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84
min_Sample	7.23	7.97	8.24	8.38	8.47	8.52	8.57	8.60	8.62	8.64
Jorion	4.90	7.01	8.26	9.12	9.76	10.27	10.67	11.00	11.28	11.52
dm	4.85	6.48	7.57	8.40	9.04	9.58	10.02	10.40	10.72	11.00
MacKinlay-Por	12.87	13.57	13.83	13.94	**13.99**	**14.01**	**14.02**	**14.03**	**14.04**	**14.04**
ew-min	10.69	9.72	9.42	9.27	9.17	9.11	9.07	9.04	9.02	9.00
Kan-Zhou	5.47	7.69	8.85	9.57	10.09	10.50	10.82	11.08	11.32	11.53
CML	10.08	11.75	12.43	12.75	12.96	13.08	13.17	13.24	13.31	13.37
CKZ	10.39	11.43	11.97	12.27	12.48	12.62	12.73	12.82	12.91	12.99

Table 8: **Portfolios with 10 Investable Assets (time-varying volatility)**
We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 10 assets. Returns are simulated from Equation (1), with $\boldsymbol{\beta}$ from Unif$[0.5, 1.5]$, market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from an NIG distribution with zero mean and skewness, standard deviation = 0.2, and excess kutosis = 4, all annualized. We draw $\boldsymbol{\alpha}$ from Unif$[-i\%, i\%]$, where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
N=10					**Panel A: α=0**					
1/N	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46
True	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**
Sample	1.27	1.87	2.32	2.68	2.98	3.25	3.50	3.74	3.95	4.15
min_True	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56	11.56
min_Sample	0.45	0.48	0.50	0.52	0.52	0.53	0.53	0.54	0.54	0.54
Jorion	1.12	1.69	2.13	2.49	2.79	3.08	3.33	3.57	3.79	3.99
dm	1.36	1.94	2.37	2.72	3.01	3.29	3.53	3.76	3.98	4.17
MacKinlay-Por	13.31	13.46	**13.49**	**13.50**	**13.51**	**13.51**	**13.51**	**13.51**	**13.51**	**13.51**
ew-min	0.57	0.54	0.54	0.54	0.55	0.55	0.55	0.55	0.55	0.56
Kan-Zhou	0.95	1.50	1.95	2.32	2.65	2.96	3.22	3.48	3.71	3.93
CML	4.12	4.93	5.41	5.65	5.83	6.00	6.14	6.35	6.50	6.67
CKZ	4.86	5.80	6.47	6.89	7.19	7.44	7.63	7.82	7.99	8.16
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46	13.46
True	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**	**14.72**
Sample	2.38	3.04	3.40	3.63	3.79	3.92	4.02	4.09	4.16	4.22
min_True	11.55	11.55	11.55	11.55	11.55	11.55	11.55	11.55	11.55	11.55
min_Sample	0.44	0.47	0.49	0.51	0.51	0.52	0.52	0.53	0.53	0.53
Jorion	2.24	2.90	3.29	3.54	3.71	3.84	3.95	4.04	4.11	4.17
dm	2.46	3.08	3.43	3.66	3.81	3.93	4.03	4.11	4.17	4.23
MacKinlay-Por	13.35	**13.51**	**13.55**	**13.56**	**13.57**	**13.57**	**13.57**	**13.57**	**13.57**	**13.57**
ew-min	0.56	0.53	0.53	0.54	0.54	0.54	0.54	0.54	0.54	0.54
Kan-Zhou	2.04	2.76	3.20	3.48	3.67	3.82	3.94	4.02	4.10	4.17
CML	4.08	4.70	4.96	5.09	5.13	5.13	5.14	5.11	5.10	5.10
CKZ	4.84	5.33	5.57	5.69	5.75	5.81	5.86	5.89	5.93	5.96
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	13.43	13.43	13.43	13.43	13.43	13.43	13.43	13.43	13.43	13.43
True	**15.50**	**15.50**	**15.50**	**15.50**	**15.50**	**15.50**	**15.50**	**15.50**	**15.50**	**15.50**
Sample	4.57	5.15	5.41	5.59	5.69	5.77	5.82	5.87	5.90	5.93
min_True	11.64	11.64	11.64	11.64	11.64	11.64	11.64	11.64	11.64	11.64
min_Sample	0.49	0.54	0.55	0.55	0.55	0.55	0.55	0.55	0.56	0.57
Jorion	4.44	5.07	5.36	5.55	5.66	5.75	5.80	5.85	5.89	5.92
dm	4.62	5.16	5.42	5.59	5.70	5.77	5.82	5.87	5.90	5.94
MacKinlay-Por	**13.51**	**13.64**	**13.70**	**13.70**	**13.70**	**13.70**	**13.69**	**13.69**	**13.69**	**13.69**
ew-min	0.61	0.60	0.59	0.57	0.57	0.57	0.57	0.57	0.57	0.58
Kan-Zhou	4.29	5.02	5.35	5.54	5.66	5.75	5.80	5.85	5.89	5.92
CML	5.15	5.60	5.76	5.88	5.95	5.99	6.00	6.02	6.04	6.06
CKZ	5.74	6.11	6.31	6.45	6.53	6.60	6.63	6.66	6.69	6.72

Table 9: **Portfolios with 25 Investable Assets (time-varying volatility)**
We report monthly Sharpe ratios (in %, for ease of exposition) for 12 portfolios formed with 25 assets. Returns are simulated from Equation (1), with $\boldsymbol{\beta}$ from Unif$[0.5, 1.5]$, market returns R_{mt} from N$(8\%, 16\%)$, idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from an NIG distribution with zero mean and skewness, standard deviation $= 0.2$, and excess kutosis $= 4$, all annualized. We draw $\boldsymbol{\alpha}$ from Unif$[-i\%, i\%]$, where $i = 0, 1, 2, 3, 4, 5$ in Panel A, B, C, D, E and F, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel A: α=0					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**	**14.43**
Sample	0.67	1.00	1.27	1.50	1.70	1.87	2.02	2.18	2.31	2.44
min_True	9.24	9.24	9.24	9.24	9.24	9.24	9.24	9.24	9.24	9.24
min_Sample	0.23	0.26	0.27	0.28	0.28	0.29	0.29	0.29	0.30	0.30
Jorion	0.62	0.95	1.22	1.44	1.65	1.81	1.96	2.12	2.25	2.38
dm	0.80	1.09	1.35	1.57	1.76	1.92	2.07	2.23	2.35	2.48
MacKinlay-Por	13.88	13.98	**14.02**	**14.02**	**14.02**	**14.02**	**14.02**	**14.02**	**14.02**	**14.02**
ew-min	0.40	0.33	0.32	0.32	0.31	0.31	0.31	0.31	0.31	0.31
Kan-Zhou	0.45	0.78	1.03	1.26	1.48	1.67	1.82	1.98	2.13	2.28
CML	4.23	5.29	5.82	6.13	6.32	6.59	6.75	6.84	7.02	7.17
CKZ	5.12	5.60	6.07	6.44	6.64	6.73	6.84	6.92	6.97	7.05
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98	13.98
True	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**	**15.18**
Sample	2.69	3.48	3.93	4.22	4.43	4.58	4.70	4.80	4.87	4.94
min_True	9.26	9.26	9.26	9.26	9.26	9.26	9.26	9.26	9.26	9.26
min_Sample	0.23	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.31	0.31
Jorion	2.61	3.42	3.89	4.19	4.41	4.55	4.68	4.78	4.86	4.93
dm	2.79	3.54	3.98	4.26	4.47	4.60	4.72	4.82	4.89	4.96
MacKinlay-Por	13.92	**14.02**	**14.08**	**14.07**	**14.08**	**14.08**	**14.08**	**14.08**	**14.08**	**14.08**
ew-min	0.40	0.35	0.33	0.32	0.32	0.32	0.32	0.32	0.33	0.33
Kan-Zhou	2.25	3.25	3.80	4.14	4.38	4.54	4.67	4.77	4.85	4.92
CML	4.40	4.87	5.01	5.12	5.16	5.18	5.23	5.27	5.29	5.32
CKZ	5.20	5.31	5.41	5.54	5.66	5.72	5.81	5.88	5.92	5.97
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99	13.99
True	**17.21**	**17.21**	**17.21**	**17.21**	**17.21**	**17.21**	**17.21**	**17.21**	**17.21**	**17.21**
Sample	6.50	7.59	8.06	8.34	8.53	8.65	8.75	8.83	8.89	8.95
min_True	9.28	9.28	9.28	9.28	9.28	9.28	9.28	9.28	9.28	9.28
min_Sample	0.24	0.29	0.30	0.29	0.30	0.31	0.32	0.32	0.33	0.33
Jorion	6.43	7.56	8.04	8.33	8.52	8.64	8.75	8.83	8.89	8.94
dm	6.58	7.63	8.09	8.36	8.55	8.66	8.77	8.84	8.90	8.96
MacKinlay-Por	**14.06**	**14.22**	**14.26**	**14.24**	**14.28**	**14.27**	**14.27**	**14.27**	**14.26**	**14.26**
ew-min	0.41	0.36	0.35	0.33	0.33	0.33	0.34	0.33	0.34	0.34
Kan-Zhou	6.21	7.52	8.03	8.32	8.52	8.64	8.75	8.83	8.89	8.94
CML	6.82	7.82	8.23	8.47	8.64	8.74	8.83	8.90	8.96	9.00
CKZ	7.37	8.19	8.58	8.83	9.00	9.11	9.21	9.28	9.34	9.39

Table 10: **Portfolios with 10 Investable Assets (Certainty-Equivalent Return)**
We report monthly Certainty-Equivalent Return (CER) (in %, for ease of exposition) for 12 portfolios formed with 10 assets. Returns are simulated from the market model: Equation (1), with $\boldsymbol{\beta}$ from Unif$[0.5, 1.5]$, market returns R_{mt} from N(8%,16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from N$(0, \sigma_i^2 I_N)$ where $\sigma_i \sim$ Unif$[0.1, 0.3]$, all annualized. We set the risk-aversion coefficient to 1. We draw $\boldsymbol{\alpha}$ from Unif$[-i\%, i\%]$, where $i = 0, 1, 2$ in Panel A, B, and C, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel A: α=0					
1/N	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
True	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**
Sample	-1192.55	-560.34	-273.79	-338.01	-22.94	-6.52	-93.78	-3.92	0.38	0.51
min_True	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
min_Sample	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
Jorion	-291.09	-98.81	-22.50	-130.79	-6.28	-2.42	-40.12	-1.84	0.41	0.48
dm	-376.16	-318.43	-344.91	-37.17	-4.52	0.16	0.03	0.36	0.48	0.51
MacKinlay-Por	-259.90	-3.79	0.45	0.49	0.52	-10.75	**0.57**	**0.57**	**0.57**	**0.57**
ew-min	-100.46	-37.29	-5.56	-91.08	-2.92	-1.74	-31.07	-1.64	0.41	0.47
Kan-Zhou	-567.77	-7156.98	-574.18	-10.33	-20.85	-3.02	-0.22	0.00	0.22	0.24
CML	-603.53	-15.68	-3.71	-2.97	-3.32	0.21	0.38	0.44	0.33	0.50
CKZ	-5.22	-13.04	-0.18	0.39	-5.31	0.44	0.47	0.45	0.49	0.46
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
True	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**
Sample	-515.26	-292.71	-4199.93	-10.69	-11.81	0.16	-0.42	0.46	-86.10	0.13
min_True	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
min_Sample	0.36	0.36	0.36	0.36	0.37	0.37	0.37	0.37	0.37	0.37
Jorion	-73.67	-86.10	-1333.76	-3.75	-3.83	0.36	0.17	0.47	-42.68	0.30
dm	-2328.02	-1731.48	-121.70	-212.70	-3.59	-0.69	-0.76	0.39	0.20	0.51
MacKinlay-Por	-46.41	-1.42	-48.03	0.19	**0.55**	**0.57**	**0.57**	**0.57**	**0.57**	**0.57**
ew-min	0.41	0.39	0.38	0.38	0.38	0.37	0.37	0.37	0.37	0.37
Kan-Zhou	-24.65	-41.26	-888.15	-2.47	-2.32	0.38	0.28	0.46	-37.40	0.31
CML	-480.28	-111.11	-6.33	-6.95	-30.94	0.03	0.45	0.17	0.44	0.49
CKZ	-469.98	-347.34	-1.22	0.12	-5.22	0.42	-0.83	0.46	0.50	0.50
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
True	**0.66**	**0.66**	**0.66**	**0.66**	**0.66**	**0.66**	**0.66**	**0.66**	**0.66**	**0.66**
Sample	-31806.63	-2514.93	-64.24	-1192.24	-107.69	-5.38	-1319.23	-4.29	-2.77	0.15
min_True	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
min_Sample	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
Jorion	-10436.55	-992.22	-14.98	-224.60	-31.84	-2.67	-418.02	-2.41	-1.17	0.30
dm	-2273.23	-6198.83	-368.35	-10.58	-15.13	-5.05	-82.18	-0.16	-2339.15	-23.83
MacKinlay-Por	-8.87	-22.77	-7.90	-0.28	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**	**0.58**
ew-min	0.41	0.39	0.38	0.38	0.37	0.37	0.37	0.37	0.37	0.37
Kan-Zhou	-5125.70	-674.64	-7.24	-69.82	-15.92	-2.38	-239.17	-2.33	-1.01	0.29
CML	-821.18	-64.80	-47.52	-184.48	-0.74	-6.29	-33.22	0.20	-143.83	-5.22
CKZ	-2521.10	-38.30	-1.99	-0.99	-743.51	0.35	-10.66	-14.80	0.53	-1.31

Table 11: **Portfolios with 10 Investable Assets (Certainty-Equivalent Return)**
We report monthly Certainty-Equivalent Return (CER) (in %, for ease of exposition) for 12 portfolios formed with 10 assets. Returns are simulated from the market model: Equation (1), with $\boldsymbol{\beta}$ from Unif$[0.5, 1.5]$, market returns R_{mt} from N(8%, 16%), idiosyncratic errors $\boldsymbol{\epsilon}$ are drawn from $\mathrm{N}(0, \sigma_i^2 I_N)$ where $\sigma_i \sim$ Unif$[0.1, 0.3]$, all annualized. We set the risk-aversion coefficient to 3. We draw $\boldsymbol{\alpha}$ from Unif$[-i\%, i\%]$, where $i = 0, 1, 2$ in Panel A, B, and C, respectively. Bold font indicates a better performance than the 1/N rule.

Rules\Months	120	240	360	480	600	720	840	960	1080	1200
					Panel A: α=0					
1/N	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
True	**0.35**	**0.35**	**0.35**	**0.35**	**0.35**	**0.35**	**0.35**	**0.35**	**0.35**	**0.35**
Sample	-4114.84	-25256.31	-3918.52	-93.20	-80.13	-28.93	-1.40	-0.44	0.04	0.12
min_True	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
min_Sample	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.23
Jorion	-1220.25	-10200.23	-1118.40	-20.71	-30.39	-7.13	-0.37	-0.04	0.20	0.23
dm	-1114.32	-8528151.63	-1929.66	-117.53	-168.64	-180.27	-20.44	-0.16	0.10	0.14
MacKinlay-Por	-108.97	-30.39	-1.45	0.24	0.25	0.28	0.28	0.29	0.29	0.29
ew-min	0.26	0.24	0.24	0.24	0.23	0.23	0.23	0.23	0.23	0.23
Kan-Zhou	-567.77	-7156.98	-574.18	-10.33	-20.85	-3.02	-0.22	0.00	0.22	0.24
CML	-3868.61	-258.11	-40814.99	-20.30	-251.52	-2.32	-17.70	0.05	0.21	0.23
CKZ	-13686.65	0.07	0.12	0.29	0.30	0.30	**0.31**	**0.31**	**0.31**	**0.31**
					Panel B: $\alpha \sim$ Unif [-1%, 1%]					
1/N	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
True	**0.36**	**0.36**	**0.36**	**0.36**	**0.36**	**0.36**	**0.36**	**0.36**	**0.36**	**0.36**
Sample	-273658.28	-1865.17	-101.59	-322.48	-66.63	-643.66	-4.49	-4.75	-48.46	-0.35
min_True	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
min_Sample	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
Jorion	-76792.53	-535.25	-27.22	-113.85	-18.22	-326.83	-1.59	-1.80	-29.98	0.06
dm	-7829.49	-3105.78	-126.04	-44.73	-78.98	-48.99	-1290.66	-11.39	-1.05	-1.95
MacKinlay-Por	-24.08	-25.86	-16.22	-7.92	0.26	0.28	0.28	0.29	0.29	0.29
ew-min	0.25	0.24	0.24	0.23	0.23	0.23	0.23	0.23	0.23	0.23
Kan-Zhou	-30234.99	-280.17	-14.53	-74.49	-8.52	-288.82	-1.12	-1.32	-29.26	0.12
CML	-277.51	-1822.06	-1452.68	-53.85	-6.69	-47.57	-3.07	-0.42	-23.11	-1.11
CKZ	-26.45	-0.11	0.13	-1.57	0.30	0.28	**0.31**	**0.32**	**0.32**	**0.32**
					Panel C: $\alpha \sim$ Unif [-2%, 2%]					
1/N	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
True	**0.38**	**0.38**	**0.38**	**0.38**	**0.38**	**0.38**	**0.38**	**0.38**	**0.38**	**0.38**
Sample	-4980.31	-922.55	-5096.08	-231.35	-92.21	-124.39	-10.17	-1.98	-168.32	-6.08
min_True	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
min_Sample	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
Jorion	-1218.93	-248.37	-2089.37	-58.23	-26.59	-44.89	-4.43	-0.83	-64.16	-3.39
dm	-1774.41	-316.88	-2353407.63	-110.58	-28.17	-6.48	-91.20	-20.11	-13.61	-22.66
MacKinlay-Por	-781.71	-1998.62	-0.01	-18.11	0.22	0.28	0.29	0.30	0.30	0.30
ew-min	0.26	0.24	0.24	0.24	0.23	0.23	0.23	0.23	0.23	0.23
Kan-Zhou	-419.26	-120.75	-1509.61	-29.52	-15.24	-29.50	-3.56	-0.71	-44.19	-3.23
CML	-283.85	-8360.01	-1077.07	-69.90	-70.99	-6.59	-3.49	-1.65	-0.34	-0.47
CKZ	-109.40	-0.68	0.20	-2.91	-0.28	**0.32**	**0.32**	**0.33**	**0.34**	**0.34**